SEQUENCING

1. **Introduction**

Sequencing is the process of scheduling jobs on machines in such a way so as to minimize the overall time, cost and resource usage thereby maximizing profits.

Let,

n Number of jobs

*−→*

m Number of machines

*−→*

Then, theoretically possible sequences = (*n*!)*m*

e.g. if n = 4 and m = 3 then

Number of sequences = (4!)3 = 16,777,216

Not possible practically to check out all sequences to find the optimal one. Hence the need of finding some suitable procedure for sequencing the jobs.

## PRIORITY SEQUENCING

When jobs compete for work centres capacity, which job should be done next? Priority sequence rules are applied to all jobs waiting in the queue. Then when the work centre be- comes open for the job, the one with the highest priority is assigned. ”Priority Sequencing” is a systematic procedure for assigning priorities to waiting jobs thereby determining the sequence in which the jobs will be performed.

## SEQUENCING PROBLEMS

The sequencing problem arises whenever there is a need for determining an optimum order of performing a number of jobs by number of facilities according to some pre-assigned order so as to optimise the output in terms of cost, time or profit.

Production of finished goods from raw materials consists of several operations to be per- formed in a given sequence. Frequently, similar operations required for several products are performed at the same work stations particularly in intermittent or batch production. Un- der such situations, it is required to select a preferred order for products passing through a work station. The problem becomes complicated when the several work stations serve many products. In such a problem, the criteria is minimum total processing time.

The general sequencing problem is stated as: There are n jobs (1, 2, 3...n) each of which must be processed through each of m machines (*m*1*, m*2*, m*3*, ......mn*) one at a time. The order of processing each job through the machines is given and also the time taken to process each job on each machine is known.

The problem is to determine the order of processing of n jobs so that the total elapsed time for all the jobs will be minimum. The general sequencing problem is to determine the opti- mal sequence from amongst (*n*!)*m* sequences that minimises the total elapsed time.

Basic Assumptions:

* 1. Only one operation is carried out on a machine at a time.
  2. Processing times are known and do not change.
  3. Processing times are independent of order of processing the job.
  4. The time required in moving jobs from one machine to another is negligibly small.
  5. Each operation once started must be performed till completion.
  6. Each preceding operation must be completed, before beginning of the next immediate operation.
  7. Only one machine of each type is available.
  8. A job is processed as soon as possible, but only in the order specified.
  9. ‘No passing rule’ is strictly followed. i.e. same order of jobs is maintained over each machine. e.g. If n jobs are to be processed on two machines A B in order A B, then each job should go to machine A first then to machine B.

*→*

1. **Sequencing Models**

All type of sequencing problems may be categorized in one of the following models:

* + Sequencing n jobs on 1 machine.
  + Sequencing n jobs on 2 machine.
  + Sequencing n jobs on 3 machine.
  + Sequencing n jobs on m machine.

# Sequencing n jobs on 1 machine

Five rules to find out optimal sequence:

* + 1. SPT rule (Shortest Processing Time).
    2. WSPT rule (Weighted Shortest Processing Time).
    3. EDD rule (Earliest Due Date).
    4. Hodgson’s Algorithm.
    5. Slack Rule.

To illustrate above rules an example is being considered:

Example: Consider 8 jobs with processing times, due dates and importance weights as shown below:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Job(i): | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Processing time (*ti*): | 5 | 8 | 6 | 3 | 10 | 14 | 7 | 3 |
| Due Date (*di*): | 15 | 10 | 15 | 25 | 20 | 40 | 45 | 50 |
| Importance Weight (*wi*): | 1 | 2 | 3 | 1 | 2 | 3 | 2 | 1 |

## SPT Rule

In this rule, job with shortest processing time is considered first then next and so on. It simply means that arranging processing time in ascending order, the job sequence could be found.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Job(i): | 4 | 8 | 1 | 3 | 7 | 2 | 5 | 6 |
| Processing time (*ti*): | 3 | 3 | 5 | 6 | 7 | 8 | 10 | 14 |

Using SPT rule the sequence of jobs will be 4-8-1-3-7-2-5-6

Completion of these jobs are at times 3, 6, 11, 17, 24, 32, 42, 56 respectively. a) Mean Flow Time = 3+6+11+17+24+32+42+56 = 191 = 23*.*9 units

8 8

b) Weighted Mean Flow Time = 1∗3+1∗6+1∗11+3∗17+2∗24+2∗32+2∗42+3∗56 = 435 = 29 units

1+1+1+3+2+2+2+3 15

* + - 1. Average in-process inventory

Average in-process inventory = 8∗3+7∗3+6∗5+5∗6+4∗7+3∗8+2∗10+1∗14 = 191 = 3*.*41 jobs

56 56

* + - 1. Waiting time for each job:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Job(i): | 4 | 8 | 1 | 3 | 7 | 2 | 5 | 6 |
| Waiting time: | 0 | 3 | 6 | 11 | 17 | 24 | 32 | 42 |

Mean Waiting time = 0+3+6+11+17+24+32+42 = 135 = 16*.*875 units

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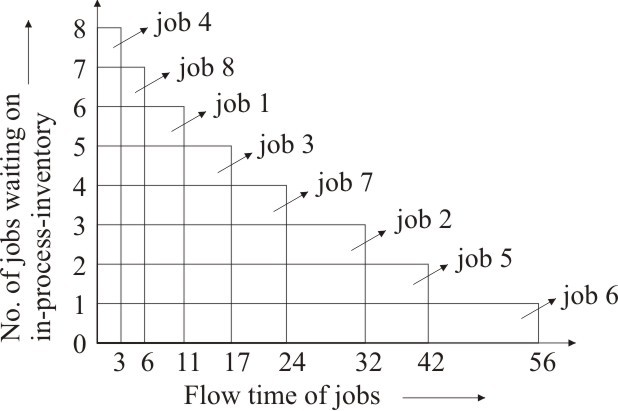


Figure 1: Average in-process inventory

* + - 1. Lateness of each job = Completion time of job - Due date of job Lateness of job 4 = 3 - 25 = -22

Lateness of job 8 = 6 - 50 = -44

Lateness of job 1 = 11 - 15 = -4

Lateness of job 3 = 17 - 15 = +2

Lateness of job 7 = 24 - 45 = -21

Lateness of job 2 = 32 - 10 = +22

Lateness of job 6 = 56 - 40 = +16 Positive Lateness is called tardiness.

Mean Lateness = −22−44−4+2−21+22+22+16 = −29 = *−*3*.*625 units

8

8

Mean tardiness = 0+0+0+2+0+22+22+16 = 62 = 7*.*8 units

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No. of tasks actually late = 4 (job no. 3,2,5,6) = No. of tardy jobs Maximum lateness = 22 units (job 5 and 2 both)

## WSPT Rule

In this rule, job with shortest ‘processing time per unit of importance’ is considered first then next and so on. It simply means that arranging ‘processing time per unit of importance’ in ascending order, the job sequence could be found.

|  |  |  |  |
| --- | --- | --- | --- |
| Job(i) | Processing time (t*i*) | Importance weight (w*i*) | *ti*  *wi* |
| 1 | 5 | 1 | 5 = 5 (VII)  1 |
| 2 | 8 | 2 | 8 = 4 (V)  2 |
| 3 | 6 | 3 | 6 = 2 (I)  3 |
| 4 | 3 | 1 | 3 = 3 (II)  1 |
| 5 | 10 | 2 | 10 = 5 (VIII)  2 |
| 6 | 14 | 3 | 14 = 4*.*7 (VI)  3 |
| 7 | 7 | 2 | 7 = 3*.*5 (IV)  2 |
| 8 | 3 | 1 | 3 = 3 (III)  1 |

Using WSPT rule the sequence of jobs will be 3-4-8-7-2-6-1-5 Respective Flowtime = 6, 9, 12, 19, 27, 41, 46, 56

Mean Flow Time = 6+9+12+19+27+41+46+56 = 216 = 27 units

8 8

b) Weighted Mean Flow Time = 3∗6+1∗9+1∗12+2∗19+2∗27+3∗41+1∗46+2∗56 = 412 = 27*.*47 units

3+1+1+2+2+3+1+2 15

Similarly, Mean Lateness = -0.5 Mean Tardiness = 10.6

No. of Tardy Jobs = 4 (job No. 2,6,1,5)] Max. Tardiness = 36 units

## EDD Rule

In this rule, jobS are sequenced in the order of increasing due dates of jobs.

Using EDD rule the sequence of jobs will be 2-1-3-5-4-6-7-8 Mean Flow Time = 32

Weighted Mean Flow Time = 31.7 Mean Lateness = +4.5 units Mean Tardiness = +5 units Maximum Lateness = 9 units

No. of late jobs = 6 (job no. 3.5.4.6.7.8) = No. of tardy jobs

## Hodgson’s Algorithm

This algorithm is aplicable only if the number of tardy jobs is more than 1. Using EDD rule number of tardy jobs could be found.

As per EDD rule, the sequence of jobs will be 2-1-3-5-4-6-7-8 with 6 tardy jobs which is more than 1 job. Hence Hodgson’s algorithm is applicable.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Job(i): | 2 | 1 | 3 | 5 | 4 | 6 | 7 | 8 |
| Processing time (*ti*): | 8 | 5 | 6 | 10 | 3 | 14 | 7 | 3 |
| Completion time (*ci*): | 8 | 13 | 19 | 29 | 32 | 46 | 53 | 56 |
| Due Date (*di*): | 10 | 15 | 15 | 20 | 25 | 40 | 45 | 50 |
| Lateness (*li* = *ci − di*): | -2 | -2 | +4 | +9 | +7 | +6 | +8 | +6 |

Since job 3 is first tardy job and is in 3rd position, examine first 3 jobs to identify the one with longest processing time. Job 2 has longest processing time of 8 units. Hence remove it and make the table again.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Job(i): | 1 | 3 | 5 | 4 | 6 | 7 | 8 |
| Processing time (*ti*): | 5 | 6 | 10 | 3 | 14 | 7 | 3 |
| Completion time (*ci*): | 5 | 11 | 21 | 24 | 38 | 45 | 48 |
| Due Date (*di*): | 15 | 15 | 20 | 25 | 40 | 45 | 50 |
| Lateness (*li* = *ci − di*): | -10 | -4 | +1 | -1 | -2 | 0 | -2 |

5th job is tardy. Since longest processing time up to 5th job i.e. from 1st, 3rd and 5th job is that of 5th job, we will remove it.

Now as per Hodgson’s algorithm, the sequence of jobs will be 1-3-4-6-7-8-2-5

Mean Lateness = 1.625 Mean Tardiness = 9

Max. Lateness = 36 units (both 2nd and 5th job) Number of tardy jobs = 2 (job 2 and 5)

## Slack Rule

Slack time = Due time of job - its processing time

This rule is based on sequencing jobs in ascending order of slack time.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Job(i): | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Processing time (*ti*): | 5 | 8 | 6 | 3 | 10 | 14 | 7 | 3 |
| Due Date (*di*): | 15 | 10 | 15 | 25 | 20 | 40 | 45 | 50 |
| Slack time (*di − ti*): | 10 | 2 | 9 | 22 | 10 | 26 | 38 | 47 |

Taking slack times in ascending order, the sequence of jobs will be 2-3-1-5-4-6-7-8 Completion times are 8,14,19,29,32,46,53 & 56

Mean tardiness = 5 units Mean Flow time = 32.1 units

Weighted Mean Flow time = 31.1 units

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Rule | Objective  (to minimize) | Mean  Flow time | Weighted  Mean Flow time | Mean  Late- ness | Maximu  Tardi- ness | m No. of  tardy jobs | Mean  tardi- ness |
| SPT | 1. mean flow time 2. mean in-   process inven- tory   1. mean waiting time 2. mean lateness | 23.9 | 29 | -3.6 | 22 | 4 | 7.8 |
| WSPT | mean flow time | 27 | 27.5 | -0.5 | 36 | 4 | 10.6 |
| EDD | maximum job  lateness | 32 | 31.7 | 4.5 | 9 | 6 | 5 |
| Hodgson | number of tardy  jobs | 29.1 | 29.9 | 1.6 | 36 | 2 | 9 |
| Slack | mean tardiness | 32.1 | 31.1 | 4.6 | 9 | 6 | 5 |

# Sequencing n jobs on 2 machine

The problem is defined as,

There are only two machines A and B. Each job is processed in the order A and B.

The processing times of n jobs (1, 2, 3 ...... n) on each of the two machines is known. Let *A*1*, A*2*........An*, are processing times on A and *B*1*, B*2*............Bn* are processing times on B. The problem is to find the order in which the n jobs are to be processed to minimise the total elapsed time (T) to complete all the n jobs.

## Procedure using Johnson’s Algorithm

Step 1: Select the smallest processing time from the given list of processing times *A*1*, A*2 *An*

and *B*1*, B*2 *Bn*.

Step 2: If the minimum processing time is *Ar* (i.e., Job number r on Machine A), do the rth job first in the sequence. If the minimum processing time is *Bs* (i.e., job number s on Machine B), do the sth job last in the sequence.

Step 3: After doing this step, (n-1) jobs are left to be sequenced. Repeat step (1) and step

(2) till all the jobs are ordered.

Step 4: Find the total processing time as per the sequence determined and also determine idle time associated with machines.

# Sequencing n jobs on 3 machine

There are three machines M1, M2 and M3. Each job has to go through three machines in the order M1, M2 and M3.

Conditions to be satisfied to solve the above problem by Johnson’s method:

1. The smallest processing time on machine M1 largest processing time on machine M2.

*≥*

1. The smallest processing time on machine M3 largest processing time on machine M2. If either or both of the above stated conditions are satisfied, the given problem can be solved by Johnson’s algorithm.

*≥*

## Procedure using Johnson’s Algorithm

Step I: Convert the three machine problem into two machine problem by introducing two fictitious machines G and H such that

*Gi* = *M* 1*i* + *M* 2*i*

*Hi* = *M* 2*i* + *M* 3*i* ( where i = 1, 2, 3, ... n)

Step II: Once the problem is converted to n job 2 machine the sequence is determined using Johnson’s algorithm for n Jobs and 2 machines.

Step III: For the optimal sequence determined, find out the minimum total elapsed time and idle times associated with machines.

## Tie breaking Rules

* 1. If there are equal smallest processing times one for each machine, place the job on machine 1, first in the sequence and one in machine 2 last in the sequence.
  2. If the equal smallest times are both for machine 1, select the job with lower processing time in machine 2 for placing first in the sequence.
  3. If the equal smallest times are both from machine 2, select the one with lower processing time in machine 1, for placing last in the sequence.

# Processing n jobs through m machines

Let there be n jobs which are to be processed through m machines *M*1*, M*2 *Mm* in the order

*M*1*, M*2 *Mm*.

Let *Tij* denote the time taken by ith job on the jth machine.

## Procedure

Step I: Find Min *Ti*1 (Minimum time for the first machine). Min *Tim* (Minimum time on the last machine).

Max (*Tij*) For j = 2,3,... m-1 and i = 1,2. n) (Maximum time on intermediate machines).

Step II: Check for the following conditions. (i) Minimum time *Ti*1 for the first machine

*M*1 *≥* Maximum time (*Tij*) on intermediate machines (*M*2 to *Mm*−1)

1. Minimum time *Tim* for the last machine *Mm ≥* Maximum time (*Tij*) on intermediate machines (*M*2 to *Mm*−1)

i.e., the minimum processing time on the machines *M*1 and *Mm* (First and last machines)

should be maximum time on any of the 2 to m-1 machines.

*≥*

Step III: If the conditions in step II are not satisfied, the problem cannot be solved by this

method, otherwise go to next step.

Step IV: Convert the n job m machine problem into n job 2 machine problem by considering two fictitious machines G and H. Such that,

*TGij* = *Ti*1 + *Ti*2 +. + *Ti*(*m*−1)

*THij* = *Ti*2 + *Ti*3 +. + *Tim*

Step V: Now obtain the sequence for n jobs using Johnson’s Algorithm.

Step VI: Determine the minimum total elapsed time and idle times associated with machines.

* + - 1. **– Queueing Theory**
    1. **Introduction:**

A flow of customers from finite or infinite population towards the service facility forms a **queue (waiting line)** an account of lack of capability to serve them all at a time. In the absence of a perfect balance between the service facilities and the customers, **waiting time** is required either for the service facilities or for the customers arrival. In general, the **queueing system** consists of one or more queues and one or more servers and operates under a set of procedures. Depending upon the server status, the incoming customer either waits at the queue or gets the turn to be served. If the server is free at the time of arrival of a customer, the customer can directly enter into the counter for getting service and then leave the system. In this process, over a period of time, the system may experience “ Customer waiting” and /or “Server idle time”

* + 1. **Queueing System:**

A queueing system can be completely described by

* + - 1. the input (arrival pattern)
      2. the service mechanism (service pattern)
      3. The queue discipline and
      4. Customer’s behaviour

**5.1.3. The input (arrival pattern)**

The input described the way in which the customers arrive and join the system. Generally, customers arrive in a more or less random manner which is not possible for prediction. Thus the arrival pattern can be described in terms of probabilities and consequently the probability distribution for **inter-arrival** times (the time between two successive arrivals) must be defined. We deal with those Queueing system in which the customers arrive in poisson process. The mean arrival rate is denoted by

.

* + 1. **The Service Mechanism:-**

This means the arrangement of service facility to serve customers. If there is infinite number of servers, then all the customers are served instantaneously or arrival and there will be no queue. If the number of servers is finite then the customers are served according to a specific order with service time a constant or a random variable. Distribution of service time follows ‘**Exponential distribution’** defined by

f(t) = e -t , t > 0

The mean Service rate is E(t) = 1/

* + 1. **Queueing Discipline:-**

It is a rule according to which the customers are selected for service when a queue has been formed. The most common disciplines are

* + - 1. First come first served – (FCFS)
      2. First in first out – (FIFO)
      3. Last in first out – (LIFO)
      4. Selection for service in random order (SIRO)
    1. **Customer’s behaviour**
       1. Generally, it is assumed that the customers arrive into the system one by one. But in some cases, customers may arrive in groups. Such arrival is called **Bulk arrival.**
       2. If there is more than one queue, the customers from one queue may be tempted to join another queue because of its smaller size. This behaviour of customers is known as **jockeying.**
       3. If the queue length appears very large to a customer, he/she may not join the queue. This property is known as

**Balking** of customers.

* + - 1. Sometimes, a customer who is already in a queue will leave the queue in anticipation of longer waiting line. This kind of departare is known as **reneging.**
    1. **List of Variables**

The list of variable used in queueing models is give below: n - No of customers in the system

C - No of servers in the system

Pn (t) – Probability of having n customers in the system at time t.

Pn - Steady state probability of having customers in the system

P0 - Probability of having zero customer in the system Lq - Average number of customers waiting in the queue.

Ls - Average number of customers waiting in the system (in the queue and in the service counters)

Wq - Average waiting time of customers in the queue.

Ws - Average waiting time of customers in the system (in the queue and in the service counters)

 - Arrival rate of customers

 - Service rate of server

 - Utilization factor of the server

 eff - Effective rate of arrival of customers M - Poisson distribution

N - Maximum numbers of customers permitted in the system. Also, it denotes the size of the calling source of the customers.

GD - General discipline for service. This may be first in first – serve (FIFS), last-in-first serve (LIFS) random order (Ro) etc.

* + 1. **Traffic intensity (or utilization factor)**

An important measure of a simple queue is its traffic intensity given by

Traffic intensity  = Mean arrival time =  (< 1)

Mean service time 

and the unit of traffic intensity is Erlang

* + 1. **Classification of Queueing models**

Generally, queueing models can be classified into six categories using Kendall’s notation with six parameters to define a model. The parameters of this notation are

P- Arrival rate distribution ie probability law for the arrival /inter – arrival time.

Q - Service rate distribution, ie probability law according to which the customers are being served.

R - Number of Servers (ie number of service stations) X - Service discipline

Y - Maximum number of customers permitted in the system. Z - Size of the calling source of the customers.

A queuing model with the above parameters is written as (P/Q/R : X/Y/Z)

* + 1. **Model 1 : (M/M/1) : (GD/  / ) Model**

In this model

1. the arrival rate follows poisson (M) distribution.
2. Service rate follows poisson distribution (M)
3. Number of servers is 1
4. Service discipline is general disciple (ie GD)
5. Maximum number of customers permitted in the system is infinite (****)
6. Size of the calling source is infinite (****)

The steady state equations to obtain, Pn the probability of having customers in the system and the values for Ls, Lq, Ws and Wq are given below.

n= 0,1,2,---- ****where  =  <1



Ls – Average number of customers waiting in the system (ie waiting in the queue and in the service station)

Pn = n (1-)

Ls = 

1- 



Lq = Ls –





= - 

1 - 

=  - (1 - ) 

1 - 

Lq

=

Aver s in the system

(in the queue and in the service station) = Ws

age waiting time of customer

2

1 - 

|  |  |  |  |
| --- | --- | --- | --- |
| = Ws = Ls = |  |  | |
|  | (1 - ) |
| = x | 1 | = | 1 |
| 1 -  |  |  | (1 - ) |

(Since  =  )

= 1

 - 

= 1

 - 

Ws = 1

 - 

Wq = Average waiting time of customers in the queue.

= Lq /  = [1 / ] [ 2 / [1- ]]

= 1 /  [ 2 / [1- ]]

=  Since  = 

 - 

Example 1:

Wq = 

 - 

The arrival rate of customers at a banking counter follows a poisson distibution with a mean of 30 per hours. The service rate of the counter clerk also follows poisson distribution with mean of 45 per hour.

1. What is the probability of having zero customer in the system ?
2. What is the probability of having 8 customer in the system ?
3. What is the probability of having 12 customer in the system ?
4. Find Ls, Lq, Ws and Wq

**Solution**

Given arrival rate follows poisson distribution with mean =30

= 30 per hour

Given service rate follows poisson distribution with mean = 45

 = 45 Per hour

Utilization factor  = /

= 30/45

= 2/3

= 0.67

1. The probability of having zero customer in the system P0 = 0 (1- )

= 1- 

= 1-0.67

= 0.33

1. The probability of having 8 customers in the system P8 = 8 (1- )

= (0.67)8 (1-0.67)

= 0.0406 x 0.33

= 0.0134

Probability of having 12 customers in the system is

|  |  |  |  |
| --- | --- | --- | --- |
|  | | P12 | = 12 (1- )  = (0.67)12 (1-0.67) |
| Ls | =  |  | = 0.0082 x 0.33 **= 0.002706**  = 0.67 |
|  | 1 -  |  | 1-0.67 |

= 0.67 = 2.03

0.33

**= 2 customers**

Lq = 2 = (0.67)2 = 0.4489

1-  1-0.67 0.33

= 1.36

**= 1 Customer**

Ws = 1 = 1 = 1

 -  45-30 15

**= 0.0666 hour**

**= 0.4467 hour**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Wq | = |  | = | 0.67 | = 0.67 |
|  |  |  -  |  | 45-30 | 15 |

Example 2 :

At one-man barbar shop, customers arrive according to poisson dist with mean arrival rate of 5 per hour and the hair cutting time was exponentially distributed with an average hair cut taking 10 minutes. It is assumed that because of his excellent reputation, customers were always willing to wait. Calculate the following:

1. Average number of customers in the shop and the average numbers waiting for a haircut.
2. The percentage of time arrival can walk in straight without having to wait.
3. The percentage of customers who have to wait before getting into the barber’s chair. Solution:-

|  |  |  |  |
| --- | --- | --- | --- |
| Given mean arrival of customer   and mean time for server  | = 5/60 | =1/12  = | 1/10 |
|  =  /  = [1/12] x 10 | = | 10 /12 |  |
|  |  | **=** | **0.833** |

* 1. Average number of customers in the system (numbers in the queue and in the service station) Ls =  / 1-  = 0.83 / 1- 0.83

= 0.83 / 0.17

= 4.88

**= 5 Customers**

* 1. The percentage of time arrival can walk straight into barber’s chair without waiting is Service utilization =  %

=  / %

= 0.833 x 100

**=83.3**

**Example 3 :**

* 1. The percentage of customers who have to wait before getting into the barber’s chair = (1-)% (1-0.833)% = 0.167 x 100

**= 16.7%**

Vehicles are passing through a toll gate at the rate of 70 per hour. The average time to pass through the gate is 45 seconds. The arrival rate and service rate follow poisson distibution. There is a complaint that the vehicles wait for a long duration. The authorities are willing to install one more gate to reduce the average time to pass through the toll gate to 35 seconds if the idle time of the toll gate is less than 9% and the average queue length at the gate is more than 8 vehicle, check whether the installation of the second gate is justified?

## Solutions:-

Arrival rate of vehicles at the toll gate  = 70 per hour Time taken to pass through the gate= 45 Seconds

Service rate  = 1 hours

45 seconds

= 3600/45 = 80

**= 80 Vehicles per hour**

 Utilization factor  = /

= 70 / 80

**= 0.875**

* + 1. Waiting no. of vehicles in the queue is Lq

Lq = 2 / 1 -  = (0.875)2

1-0.875

= 0.7656

0.125

**= 6.125**

**= 6 Vehicles**

* + 1. Revised time taken to pass through the gate =30 seconds

The new service rate after installation of an additional gate = 1 hour/35 Seconds = 3600/35

**= 102.68 Vehicles / hour**

 Utilization factor  =  /  = 70 / 102.86

**= 0.681**

Percentage of idle time of the gate = (1-)%

= (1-0.681)%

= 0.319%

= 31.9

**= 32%**

This idle time is not less than 9% which is expected.

Therefore the installation of the second gate is not justified since the average waiting number of vehicles in the queue is more than 8 but the idle time is not less than 32%. Hence idle time is far greater than the number. of vehicles waiting in the queue.

* + 1. **Second Model (M/M/C) : (GD/ /  )Model**

The parameters of this model are as follows:

1. Arrival rate follows poisson distribution
2. Service rate follows poisson distribution
3. No of servers is C’.
4. Service discipline is general discipline.
5. Maximum number of customers permitted in the system is infinite

Then the steady state equation to obtain the probability of having n customers in the system is Pn =  n Po , o  n  C

~~n!~~

=  n Po for n > c Where  / c < 1 C n-c C!

Where [ / c] < 1 as  =  / 

C-1

 P0 ={[ n/n!] + c / (c! [1 - /c])}-1

n = 0

where c! = 1 x 2 x 3 x upto C

Lq = [ c+1 / [c-1! (c -  )] ] x P0

= (c Pc) / (c -  )2

Ls = Lq + and Ws = Wq + 1 / 

Wq = Lq / 

Under special conditions Po = 1 -  and Lq =  C+! / c 2 Where  <1 and Po = (C-) (c – 1)! / c c

and Lq =  / (c- ), where  / c < 1

***Example 1:***

At a central warehouse, vehicles are at the rate of 24 per hour and the arrival rate follows poisson distribution. The unloading time of the vehicles follows exponential distribution and the unloading rate is 18 vehicles per hour. There are 4 unloading crews. Find

1. Po and P3
2. Lq, Ls, Wq and Ws

Solution:

Arrival rate  = 24 per hour Unloading rate  = 18 Per hour No. of unloading crews C=4

 =  /  = 24 / 18**=1.33**

C-1

(i) P0 ={[ n/n!] + c / (c! [1 - /c])}-1

n = 0

3

={[ (1.33)n/n!]+ (1.33)4 /(4! [1 - (1.33)/ 4])}-1

n = 0

={ (1.33)0 / = 0! + (1.33)1 / 1! + (1.33)2 / 2! + (1.33)3 / 3! +

(1.33)4 / 24! [1 - (1.33)/ 4] }-1

=[1 + 1.33 + 0.88 + 0.39 + 3.129/16.62] -1

=[3.60 + 0.19]-1 = [3.79]-1

**= 0.264**

We know Pn = ( n / n!) Po for 0  n  c

 P3 = ( 3 / 3!) Po Since 0  3  4

= [(1.33)3 / 6 ] x 0.264

= 2.353 x 0.044

**= 0.1035**

* 1. Lq = C+1 X P0

|  |  |  |
| --- | --- | --- |
| = | (C – 1)! (C-)2 (1.33)5 | X 0.264 |
|  | 3! X (4 – 1.33)2 |  |
| = | (4.1616) X  6 X (2.77)2 | 0.264 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | | = | (4.1616) X | 0.264 |
|  | 46 .0374 |  |
| = | 1.099 / 46.0374 |  |
| = | 0.0239 |  |
| Ls | = | **=**  Lq +  | **0.0239 Vehicles**  = | 0.0239 + 1.33 |
| Wq | = | Lq /  | **=**  =  **=** | **1.3539 Vehicles**  0.0239 /24  **0.000996 hrs** |

Ws = Wq + 1 /  = 0.000996 + 1/18

= 0.000996 + 0.055555

= 0.056551 hours.

## Example 2 :-

A supermarket has two girls ringing up sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if the people arrive in poisson fashion at the rate of 10 per hour

* + 1. What is the probability of having to wait for service?
    2. What is the expected percentage of idle time for each girl?
    3. If a customer has to wait, what is the expected length of his waiting time?

## Solution:-

C-1

P0 ={[ n/n!] + c / (c! [1 - /c])}-1

n = 0

Where  =  /   given arrival rate = 10 per hour

 = 10 / 60 = 1 / 6 per minute Service rate = 4 minutes

  = 1 / 4 person per minute Hence  =  /  = (1 / 6) x 4 = 2 / 3

**= 0.67**

1

P0 ={[ n/n!]+(0.67)2 / (2! [1 - 0.67/2])}-1

n = 0

=[1 + (  / 1!) ] + 0.4489 / (2 – 0.67)]-1

=[1 + 0.67 + 0.4489 / (1.33)]-1

=[1 + 0.67 + 0.34]-1

=[ 2.01]-1

**= 1 / 2**

The Probability of having to wait for the service is

## P (w > 0)

=  c X P0 c! [1 -  /c]

= 0.67 2X (1 / 2)

2! [1 – 0.67 /2]

= 0.4489 / 2.66

**= 0.168**

1. The probability of idle time for each girl is

= 1- P (w > 0)

= 1-1/3

**= 2/3**

 Percentage of time the service remains idle = 67% approximately

1. The expected length of waiting time (w/w>0)

= 1 / (c  - )

= 1 / [(1 / 2) – (1 / 6) ]

**= 3 minutes**

***Examples 3 :***

A petrol station has two pumps. The service time follows the exponential distribution with mean 4 minutes and cars arrive for service in a poisson process at the rate of 10 cars per hour. Find the probability that a customer has to wait for service. What proportion of time the pump remains idle?

## Solution: Given C=2 The arrival rate = 10 cars per hour.

  = 10 / 60 = 1 / 6 car per minute Service rate = 4 minute per cars.

Ie  = ¼ car per minute.

 =  /  = (1/6) / (1/4)

= 2 / 3

**= 0.67**

Proportion of time the pumps remain busy

=  / c = 0.67 / 2

= 0.33

= 1 / 3

 The proportion of time, the pumps remain idle

=1 – proportion of the pumps remain busy

= 1-1 / 3 = 2 / 3

C-1

P0 ={[ n/n!] + c / (c! [1 - /c])}-1

n = 0

=[ ( 0.67)0 / 0!) + ( 0.67)1 / 1!) + ( 0.67)2 / 2!)[1- ( 0.67 / 2)1 ]-1

=[1 + 0.67 + 0.4489 / (1.33)]-1

=[1 + 0.67 + 0.33]-1

=[ 2]-1

**=1 / 2**

Probability that a customer has to wait for service

= p [w>0]

=  c x P0 = (0.67)2 x 1/2

[c [1 -  / c] [2![1 – 0.67/2]

= 0.4489 = 0.4489

1.33x2 2.66

**= ~~0.1688~~**

* 1. **Simulation :**

Simulation is an experiment conducted on a model of some system to collect necessary information on the behaviour of that system.

* + 1. **Introduction :**

## The representation of reality in some physical form or in some form of Mathematical equations are called Simulations .

Simulations are imitation of reality.

**For example :**

* + - 1. Children cycling park with various signals and crossing is a simulation of a read model traffic system
      2. Planetarium
      3. Testing an air craft model in a wind tunnel.
    1. **Need for simulation :**

Consider an example of the queueing system, namely the reservation system of a transport corporation. The elements of the system are booking counters (servers) and waiting customers (queue). Generally the arrival rate of customers follow a Poisson distribution and the service time follows exponential distribution. Then the queueing model (M/M/1) : (GD/ / ) can be used to find the standard results. But in reality, the following combinations of distributions my exist.

* + - 1. Arrived rate does not follow Poisson distribution, but the service rate follows an exponential distribution.
      2. Arrival rate follows a Poisson distribution and the service rate does not follow exponential distribution.
      3. Arrival rate does not follows poisson distribution and the service time also does not follow exponential distribution. In each of the above cases, the standard model (M/M/1) : (G/D/ / ) cannot be used. The last resort to find the solution for such a queueing problem is to use simulation.
    1. **Some advantage of simulation :**
       1. Simulation is Mathematically less complicated
       2. Simulation is flexible
       3. It can be modified to suit the changing environments.
       4. It can be used for training purpose
       5. It may be less expensive and less time consuming in a quite a few real world situations.
    2. **Some Limitations of Simulation :**
       1. Quantification or Enlarging of the variables maybe difficult.
       2. Large number of variables make simulations unwieldy and more difficult.
       3. Simulation may not. Yield optimum or accurate results.
       4. Simulation are most expensive and time consuming model.
       5. We cannot relay too much on the results obtained from simulation models.
    3. **Steps in simulation :**
       1. Identify the measure of effectiveness.
       2. Decide the variables which influence the measure of effectiveness and choose those variables, which affects the measure of effectiveness significantly.
       3. Determine the probability distribution for each variable in step 2 and construct the cumulative probability distribution.
       4. Choose an appropriate set of random numbers.
       5. Consider each random number as decimal value of the cumulative probability distribution.
       6. Use the simulated values so generated into the formula derived from the measure of effectiveness.
       7. Repeat steps 5 and 6 until the sample is large enough to arrive at a satisfactory and reliable decision.
    4. **Uses of Simulation**

Simulation is used for solving 1.Inventory Problem

* + - * 1. Queueing Problem
        2. Training Programmes etc.

***Example :***

Customers arrive at a milk booth for the required service. Assume that inter – arrival and service time are constants and given by 1.5 and 4 minutes respectively. Simulate the system by hand computations for 14 minutes.

1. What is the waiting time per customer?
2. What is the percentage idle time for the facility? (Assume that the system starts at t = 0)

***Solution :***

**First customer** arrives at the service center at t = 0

 His departure time after getting service = 0 + 4 = 4 minutes.

**Second customer** arrives at time t = 1.5 minutes

##  he has to wait = 4 – 1.5 = 2.5 minutes.

**Third customer** arrives at time t = 3 minutes

 he has to wait for = 8-3 = 5 minutes

**Fourth customer** arrives at time t = 4.5 minutes and he has to wait for 12 – 4.5 = 7.5 minutes.

During this 4.5 minutes, the first customer leaves in 4 minutes after getting service and the second customer is getting service.

**Fifth customer** arrives at t = 6 minutes

 he has to wait 14 – 6 = 8 minutes

**Sixth customer** arrives at t = 7.5 minutes

he has to wait 14 – 7.5 = 6.5 minutes

**Seventh customer** arrives at t = 9 minutes

he has to wait 14 – 9 = 5 minutes

During this 9 minutes the second customer leaves the service in 8th minute and third person is to get service in 9th minute.

**Eighth customer** arrives at t = 10.5 minutes

he has to wait 14 – 10.5 = 3.5 minutes

**Nineth customer** arrives at t = 12 minutes

he has to wait 14 – 12 = 2 minutes

But by 12th minute the third customer leaves the Service

**10th Customer** arrives at t = 13.5 minutes

he has to wait 14-13.5 = 0.5 minute From this simulation table it is clear that

1. Average waiting time for 10 customers

= 2.5+5+7.5+8+6.5+5.0+3.5+2+0.5

~~10~~

= 40.5 = 4.05

~~10~~

1. Average waiting time for 9 customers who are in waiting for service 40.5 = 4.5 minutes. 9

But the average service time is 4 minutes which is less that the average waiting time, the percentage of idle time for service = 0%

* 1. **Replacement models**
     1. **Introduction:**

The replacement problems are concerned with the situations that arise when some items such as men, machines and usable things etc need replacement due to their decreased efficiency, failure or breakdown. Such decreased efficiency or complete breakdown may either be gradual or all of a sudden.

If a firm wants to survive the competition it has to decide on whether to replace the out dated equipment or to retain it, by taking the cost of maintenance and operation into account. There are two basic reasons for considering the replacement of an equipment.

They are

* + - 1. Physical impairment or malfunctioning of various parts.
      2. Obsolescence of the equipment.

The physical impairment refers only to changes in the physical condition of the equipment itself. This will lead to decline in the value of service rendered by the equipment, increased operating cost of the equipments, increased maintenance cost of the equipment or the combination of these costs. Obsolescence is caused due to improvement in the existing Tools and machinery mainly when the technology becomes advanced therefore, it becomes uneconomical to continue production with the same equipment under any of the above situations. Hence the equipments are to be periodically replaced. Some times, the capacity of existing facilities may be in adequate to meet the current demand. Under such cases, the following two alternatives will be considered.

1. Replacement of the existing equipment with a new one
2. Argument the existing one with an additional equipments.
   * 1. **Type of Maintenance**

Maintenance activity can be classified into two types

* + - 1. Preventive Maintenance
      2. Breakdown Maintenance

Preventive maintenance (PN) is the periodical inspection and service which are aimed to detect potential failures and perform minor adjustments a requires which will prevent major operating problem in future. Breakdown maintenance is the repair which is generally done after the equipment breaks down. It is offer an emergency which will have an associated penalty in terms of increasing the cost of maintenance and downtime cost of equipment, Preventive maintenance will reduce such costs up-to a certain extent . Beyond that the cost of preventive maintenance will be more when compared to the cost of the breakdown maintenance.

Total cost = Preventive maintenance cost + Breakdown maintenance cost.

This total cost will go on decreasing up-to P with an increase in the level of maintenance up-to a point, beyond which the total cost will start increasing from P. The level of maintenance corresponding to the minimum total cost at P is the Optional level of maintenance this concept is illustrated in the follows diagram

## N

**M**

### The points M and N denote optimal level of maintenance and optimal cost respectively

* + 1. **Types of replacement problem**

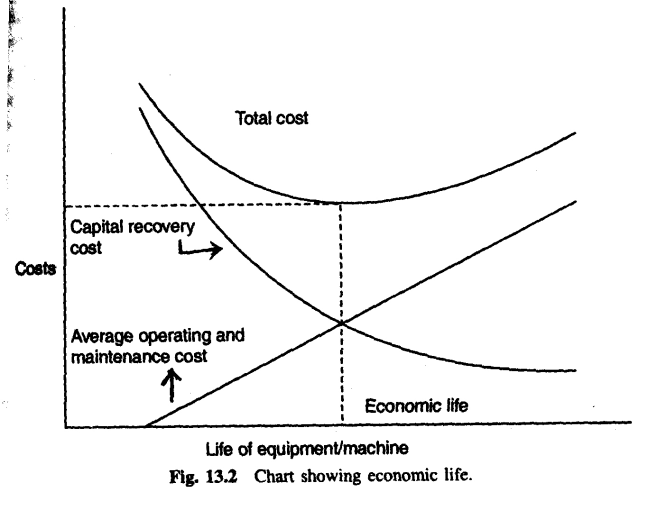
The replacement problem can be classified into two categories.

1. Replacement of assets that deteriorate with time (replacement due to gradual failure, due to wear and tear of the components of the machines) This can be further classified into the following types.
   1. Determination of economic type of an asset.
   2. Replacement of an existing asset with a new asset.
2. simple probabilistic model for assets which will fail completely (replacement due to sudden failure).
   * 1. **Determination of Economic Life of an asset**

Any asset will have the following cost components

1. Capital recovery cost (average first cost), Computed form the first cost (Purchase price) of the asset.
2. Average operating and maintenance cost.
3. Total cost which is the sum of capital recovery cost (average first cost) and average operating and maintenance cost.

## A typical shape of each of the above cost with respect to

**life of the asset is shown below**

From figure, when the life of the machine increases, it is clear that the capital recovery cost (average first cost) goes on decreasing and the average operating and maintenance cost goes on increasing. From the beginning the total cost goes on decreasing upto a particular life of the asset and then it starts increasing. The point P were the total cost in the minimum is called the Economic life of the asset. To solve problems under replacement, we consider the basics of interest formula.

Present worth factor denoted by (P/F, i,n). If an amount P is invested now with amount earning interest at the rate i per year, then the future sum (F) accumulated after n years can be obtained.

P - Principal sum at year Zero

F - Future sum of P at the end of the nth year i - Annual interest rate

n - Number of interest periods.

Then the formula for future sum F = P ( 1 + i ) n

P = F/(1 +i)n = Fx (present worth factor)

If A is the annual equivalent amount which occurs at the end of every year from year one through n years is given by

|  |  |  |
| --- | --- | --- |
| A | =  = | P x i (1 +i)n (1 +i)n - 1  P ( A / P, i, n ) |
|  | = | P x equal payment series capital recovery factor |
| ***Example:*** |  |  |

A firm is considering replacement of an equipment whose first cost is Rs. 1750 and the scrap value is negligible at any year. Based on experience, it is found that maintenance cost is zero during the first year and it increases by Rs. 100 every year thereafter.

1. When should be the equipment replaced if
   1. i = 0%

b) i = 12%

### Solution :

Given the first cost = Rs 1750 and the maintenance cost is Rs. Zero during the first years and then increases by Rs. 100 every year thereafter. Then the following table shows the calculation.

## Calculations to determine Economic life

**(a) First cost Rs. 1750 Interest rate = 0%**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **End of year (n)** | **Maintenan ce cost at end of year** | **Summation of maintenanc**  **e** | **Average cost of maintenance**  **through the** | **Average first cost if**  **replaced at**  **the given** | **Average total cost through the**  **given year** |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | **Cost** | **given year** | **year and** |  |
| **A** | **B (Rs)** | **C (Rs)** | **D (in Rs)** | **E (Rs)** | **F (Rs)** |
|  |  | **C = B** | **C/A** | **1750**  **A** | **D + E** |
| 1 | 0 | 0 | 0 | 1750 | 1750 |
| 2 | 100 | 100 | 50 | 875 | 925 |
| 3 | 200 | 300 | 100 | 583 | 683 |
| 4 | 300 | 600 | 150 | 438 | 588 |
| 5 | 400 | 1000 | 200 | 350 | 550 |
| **6** | **500** | **1500** | **250** | **292** | **542** |
| 7 | 600 | 2100 | 300 | 250 | 550 |
| 8 | 700 | 2800 | 350 | 219 | 569 |

The value corresponding to any end-of-year (n) in Column F represents the average total cost of using the equipment till the end of that particulars year.

In this problem, the average total cost decreases till the end of the year 6 and then it increases.

Hence the optimal replacement period is 6 years ie the economic life of the equipment is 6 years.

## (e) When interest rate i = 12%

When the interest rate is more than 0% the steps to get the economic life are summarized in the following table.

## Calculation to determine Economic life

**First Cost = Rs. 1750 Interest rate = 12%**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **En d of ye ar (n)** | **Mai nten ance cost at end of**  **year s** | **(P/F,12v,n)** | **Present worth as beginning of years**  **1 of**  **maintenanc e costs** | **Summation of present worth of maintenanc e costs**  **through the given year** | **Present simulator maintena nce cost**  **and first cost** | **(A/P,**  **12%,n)**  **=i (1+i)n (1+i)n-1**  **G** | **Annual equipment total cost through the giver year** |
| **A** | **B** | **C** | **D** | **E** | **F** | **G** | **H** |
|  | **B (iR)** | **C =**  **1 (1+12/100) n** | **BxC** | **D** | **E+ Rs. 1750** |  | **FxG** |
| 1 | 0 | 0.8929 | 0 | 0 | 1750 | 1.1200 | 1960 |
| 2 | 100 | 0.7972 | 79.72 | 79.72 | 1829.72 | 0.5917 | 1082.6 |
| 3 | 200 | 0.7118 | 142.36 | 222.08 | 1972.08 | 0.4163 | 820.9 |
| 4 | 300 | 0.6355 | 190.65 | 412.73 | 2162.73 | 0.3292 | 711.9 |
| 5 | 400 | 0.5674 | 226.96 | 639.69 | 2389.69 | 0.2774 | 662.9 |
| 6 | 500 | 0.5066 | 253.30 | 892.99 | 2642.99 | 0.2432 | 642.7 |
| 7 | 600 | 0.4524 | 271.44 | 1164.43 | 2914.430 | 0.2191 | 638.5 |
| 8 | 700 | 0.4039 | 282.73 | 1447.16 | 3197.16 | 0.2013 | 680.7 |

Identify the end of year for which the annual equivalent total cost is minimum in column. In this problem the annual equivalent total cost is minimum at the end of year hence the economics life of the equipment is 7 years.

* + 1. **Simple probabilistic model for items which completely fail**

Electronic items like bulbs, resistors, tube lights etc. generally fail all of a sudden, instead of gradual failure. The sudden failure of the item results in complete breakdown of the system. The system may contain a collection of such items or just an item like a single tube-light. Hence we use some replacement policy for such items which would minimize the possibility of complete breakdown. The following are the replacement policies which are applicable in these cases.

## Individual replacement policy :

Under this policy, each item is replaced immediately after failure.

## Group replacement policy :

Under group replacement policy, a decision is made with regard the replacement at what equal internals, all the item are to be replaced simultaneously with a provision to replace the items individually which fail during the fixed group replacement period.

Among the two types of replacement polices, we have to decide which replacement policy we have to follow. Whether individual replacement policy is better than group replacement policy. With regard to economic point of view. To decide this, each of the replacement policy is calculated and the most economic one is selected for implementation.

### Exercise :

* 1. List and explain different types of maintenance
  2. Discuss the reasons for maintenance.
  3. Distinguish between breakdown maintenance and preventive maintenance.
  4. Distinguish between individual and group replacement polices.
  5. A firm is considering replacement of an equipment whose first cost is Rs.4000 and the scrap value is negligible at the end of any year. Based on experience, it has been found that the maintenance cost is zero during the first year and it is Rs.1000 for the second year. It increase by Rs.300 every years thereafter.
     1. When should the equipment be replace if i = 0%
     2. When should the equipment be replace if i = 12%

## Ans . a) 5 years b) 5 years

* 1. A company is planning to replace an equipment whose first cost is Rs.1,00,000. The operating and maintenance cost of the equipment during its first year of operation is Rs.10,000 and it increases by Rs. 2,000 every year thereafter. The release value of the equipment at the end of the

first year of its operation is Rs.65,000 and it decreases by Rs.10,000 every year thereafter. Find the economic life of the equipment by assuming the interest rate as 12%.

## [Ans : Economic life = 13 years and the corresponding annual equivalent cost = Rs. 34,510]

* 1. The following table gives the operation cost, maintenance cost and salvage value at the end of every year of machine whose purchase value is Rs. 12,000. Find the economic life of the machine assuming.
     1. The interest rate as 0%
     2. The interest rate as 15%

|  |  |  |  |
| --- | --- | --- | --- |
| **End of year** | **Operation cost at**  **the end of year (Rs)** | **Maintenance cost**  **at the end of year (Rs)** | **Salvage value at the end of year (Rs)** |
| 1 | 2000 | 2500 | 8000 |
| 2 | 3000 | 3000 | 7000 |
| 3 | 4000 | 3500 | 6000 |
| 4 | 5000 | 4000 | 5000 |
| 5 | 6000 | 4500 | 4000 |
| 6 | 7000 | 5000 | 3000 |
| 7 | 8000 | 5500 | 2000 |
| 8 | 9000 | 6000 | 1000 |

## Ans :

1. Economic life of the machine = 2 years
2. Economic life of the machine = 2 years