# UNIT 5

**NETWORK PROBLEMS**

**Introduction**

A network consists of several destinations or jobs which are linked with one another. A manager will have occasions to deal with some network or other. Certain problems pertaining to networks are taken up for consideration in this unit.

**SHORTEST PATH PROBLEM**

**LESSON OUTLINE**

* The description of a shortest path problem.
* The determination of the shortest path.

# LEARNING OBJECTIVES

## After reading this lesson you should be able to

* understand a shortest path problem
* understand the algorithm for a shortest path problem
* work out numerical problems

# THE PROBLEM

Imagine a salesman or a milk vendor or a post man who has to cover certain previously earmarked places to perform his daily routines. It is assumed that all the places to be visited by him are connected well for a suitable mode of transport. He has to cover all the locations. While doing so, if he visits the same place again and again on the same day, it will be a loss of several resources such as time, money, etc. Therefore he shall place a constraint upon himself not to visit the same place again and again on the same day. He shall be in a position to determine a route which would enable him to cover all the locations, fulfilling the constraint.

The shortest route method aims to find how a person can travel from one location to another, keeping the total distance traveled to the minimum. In other words, it seeks to identify the shortest route to a series of destinations.

# EXAMPLE

Let us consider a real life situation involving a shortest route problem.

A leather manufacturing company has to transport the finished goods from the factory to the store house. The path from the factory to the store house is through certain

intermediate stations as indicated in the following diagram. The company executive wants to identify the path with the shortest distance so as to minimize the transportation cost. The problem is to achieve this objective.

95 Store house

2 4

Factory 40 40

35 65 70 6

40

1

100

5

20

3

Linkages from Factory to Store house

The shortest route technique can be used to minimize the total distance from a node designated as the

**starting node or origin** to another node designated as the **final node**.

In the example under consideration, the origin is the factory and the final node is the store house.

# STEPS IN THE SHORTEST ROUTE TECHNIQUE

The procedure consists of starting with a set containing a node and enlarging the set by choosing a node in each subsequent step.

**Step 1:**

First, locate the origin. Then, find the node nearest to the origin. Mark the distance between the origin and the nearest node in a box by the side of that node.

In some cases, it may be necessary to check several paths to find the nearest node.

**Step 2:**

Repeat the above process until the nodes in the entire network have been accounted for. The last distance placed in a box by the side of the ending node will be the distance of the shortest route. We note that the distances indicated in the boxes by each node constitute the shortest route to that node. These distances are used as intermediate results in determining the next nearest node.

# SOLUTION FOR THE EXAMPLE PROBLEM

Looking at the diagram, we see that node 1 is the origin and the nodes 2 and 3 are neighbours to the origin. Among the two nodes, we see that node 2 is at a distance of 40 units from node 1 whereas node 3 is at a distance of 100 units from node 1. The minimum of {40, 100} is 40. Thus, the node nearest to the origin is node 2, with a distance of 40 units. So, out of the two nodes 2 and 3, we select node 2. We form a set of nodes {1, 2} and construct a path connecting the node 2 with node 1 by a thick line and mark the distance of 40 in a box by the side of node 2. This first iteration is shown in the following diagram.

40

95 Store house

Factory

2

4

40 40

6

35

65

70

1

40

100 5

3

20

# ITERATION No. 1

Now we search for the next node nearest to the set of nodes {1, 2}. For this purpose, consider those nodes which are neighbours of either node 1 or node 2. The nodes 3, 4 and 5 fulfill this condition. We calculate the following distances.

The distance between nodes 1 and 3 = 100.

The distance between nodes 2 and 3 = 35.

The distance between nodes 2 and 4 = 95.

The distance between nodes 2 and 5 = 65.

Minimum of {100, 35, 95, 65} = 35.

Therefore, node 3 is the nearest one to the set {1, 2}. In view of this observation, the set of nodes is enlarged from {1, 2} to {1, 2, 3}. For the set {1, 2, 3}, there are two possible paths, viz. Path 1 → 2 → 3 and Path 1 →

3 → 2. The Path 1 → 2 → 3 has a distance of 40 + 35 = 75 units while the Path 1 → 3 → 2 has a distance of

100 + 35 = 135 units.

Minimum of {75, 135} = 75. Hence we select the path 1 → 2 → 3 and display this path by thick edges. The distance 75 is marked in a box by the side of node 3. We obtain the following diagram at the end of Iteration No. 2.

40

95 Store house

2

4

40 40

6

35

65

70

1

40

100

5

3

20

Factory

75

# ITERATION No. 2

**REPEATING THE PROCESS**

We repeat the process. The next node nearest to the set {1, 2, 3} is either node 4 or node 5.

Node 4 is at a distance of 95 units from node 2 while node 2 is at a distance of 40 units from node 1. Thus, node 4 is at a distance of 95 + 40 = 135 units from the origin.

As regards node 5, there are two paths viz. 2 → 5 and 3 → 5, providing a link to the origin. We already know the shortest routes from nodes 2 and 3 to the origin. The minimum distances have been indicated in boxes near these nodes. The path 3 → 5 involves the shortest distance. Thus, the distance between nodes 1 and 5 is 95 units (20 units between nodes 5 and 3 + 75 units between node 3 and the origin). Therefore, we select node 5 and enlarge the set from {1, 2, 3} to {1, 2, 3, 5}. The distance 95 is marked in a box by the side of node 5. The following diagram is obtained at the end of Iteration No. 3.

40

95 Store house

1

Factory 100

2 4

40 40

6

35 65 70

40

5

3

95

20

75

# ITERATION No. 3

Now 2 nodes remain, viz., nodes 4 and 6. Among them, node 4 is at a distance of 135 units from the origin (95 units from node 4 to node 2 + 40 units from node 2 to the origin). Node 6 is at a distance of 135 units from the origin (40 + 95 units). Therefore, nodes 4 and 6 are at equal distances from the origin. If we choose node 4, then travelling from node 4 to node 6 will involve an additional distance of 40 units. However, node 6 is the ending node. Therefore, we select node 6 instead of node 4. Thus the set is enlarged from {1, 2, 3, 5} to {1, 2, 3, 5, 6}. The distance 135 is marked in a box by the side of node 6. Since we have got a path beginning from the start node and terminating with the stop node, we see that the solution to the given problem has been obtained. We have the following diagram at the end of Iteration No. 4.

40

95 Store house

2 4

40 40

6

35 65 70

135

1 40

Factory 100 5

3

95

20

75

# ITERATION No. 4

**MINIMUM DISTANCE**

Referring to the above diagram, we see that **the shortest route** is provided by the path 1 **→** 2

**→** 3 **→** 5 **→** 6 with a minimum distance of 135 units.

# QUESTIONS

1. Explain the shortest path problem.
2. Explain the algorithm for a shortest path problem
3. Find the shortest path of the following network:

30

3 5

40

40

1

30

50

30

45

25

6

35

2

4

1. Determine the shortest path of the following network:

2

16

5

7

9

7

1

4

15

6

8 4

25

3

# LESSON OUTLINE

**MINIMUM SPANNING TREE PROBLEM**

* The description of a minimum spanning tree problem.
* The identification of the minimum spanning tree.

# LEARNING OBJECTIVES

## After reading this lesson you should be able to

* understand a minimum spanning tree problem
* understand the algorithm for minimum spanning tree problem
* locate the minimum spanning tree
* carry out numerical problems

**Tree:** A minimally connected network is called a tree. If there are n nodes in a network, it will be a tree if the number of edges = *n-1.*

# Minimum spanning tree algorithm

**Problem** : Given a connected network with weights assigned to the edges, it is required to find out a tree whose nodes are the same as those of the network.

The weight assigned to an edge may be regarded as the distance between the two nodes with which the edge is incident.

# Algorithm:

The problem can be solved with the help of the following algorithm. The procedure consists of selection of a node at each step.

**Step 1**: First select any node in the network. This can be done arbitrarily. We will start with this node.

**Step 2**: Connect the selected node to the nearest node.

**Step 3**: Consider the nodes that are now connected. Consider the remaining nodes. If there is no node remaining, then stop. On the other hand, if some nodes remain, among them find out which one is nearest to the nodes that are already connected. Select this node and go to Step 2.

Thus the method involves the repeated application of Steps 2 and 3. Since the number of nodes in the given network is finite, the process will end after a finite number of steps. The algorithm will terminate with step 3.

# How to break ties:

While applying the above algorithm, if some nodes remain in step 3 and if there is a tie in the nearest node, then the tie can be broken arbitrarily.

As a consequence of tie, we may end up with more than one optimal solution.

# Problem 1:

Determine the minimum spanning tree for the following network.

**60** 5

2 **70**

# 60 60 80

**100** 7

1 3

# 40 120 50

8

201

4 6

# Solution:

**80 50 60**

**30**

**90**

**Step 1:** First select node 1. (This is done arbitrarily)

**Step 2:** We have to connect node 1 to the nearest node. Nodes 2, 3 and 4 are adjacent to node

1. They are at distances of 60, 40 and 80 units from node 1. Minimum of {60, 40, 80} = 40. Hence the shortest distance is 40. This corresponds to node 3. So we connect node 1 to node 3 by a thick line. This is Iteration No. 1.

**60**

5

2

**70**

**60 60 80**

**100**

7

1

**40**

3

**120**

**50**

**80**

**50**

**60**

8

**30**

4

**90**

6

# Iteration No. 1

**Step 3:** Now the connected nodes are 1 and 3. The remaining nodes are 2, 4, 5, 6, 7 and 8. Among them, nodes 2 and 4 are connected to node 1. They are at distances of 60 and 80 from node 1. Minimum of {60, 80} = 60. So the shortest distance is 60. Next, among the nodes 2, 4, 5, 6, 7 and 8, find out which nodes are connected to node 3. We find that all of them are connected to node 3. They are at distances of 60, 50, 80, 60, 100 and 120 from node 3.

Minimum of {60, 50, 80, 60, 100, 120} = 50. Hence the shortest distance is 50.

Among these nodes, it is seen that node 4 is nearest to node 3.

Now we go to Step 2. We connect node 3 to node 4 by a thick line. This is Iteration

No.2.

**60**

5

2

**70**

**60 60 80**

**100**

7

1

3

**40**

**120**

**50**

**80**

**50**

**60**

8

**30**

4

**90**

6

# Iteration No. 2

Next go to step 3.

Now the connected nodes are 1, 3 and 4. The remaining nodes are 2, 5, 6, 7 and 8.

Node 2 is at a distance of 60 from node 1. Nodes 5, 6, 7 and 8 are not adjacent to node 1. All

of the nodes 2, 5, 6, 7 and 8 are adjacent to node 3. Among them, nodes 2 and 6 are nearer to node 3, with equal distance of 60.

Node 6 is adjacent to node 4, at a distance of 90. Now there is a tie between nodes 2 and 6. The tie can be broken arbitrarily. So we select node 2. Connect node 3 to Node 2 by a thick line. This is Iteration No. 3.

**60**

5

2

**70**

**60 60 80**

**100**

7

1

**40**

3

**120**

**50**

**80**

**50**

**60**

8

**30**

4

**90**

6

# Iteration No. 3

We continue the above process.

Now nodes 1, 2, 3 and 4 are connected. The remaining nodes are 5, 6, 7 and 8. None of them is adjacent to node 1. Node 5 is adjacent to node 2 at a distance of 60. Node 6 is at a distance of 60 from node 3. Node 6 is at a distance of 90 from node 4. There is a tie between

nodes 5 and 6. We select node 5. Connect node 2 to node 5 by a thick line. This is Iteration No. 4.

**60**

5

2

**70**

**60 60 80**

**100**

7

1

**40**

3

**120**

**50**

**80**

**50**

**60**

8

**30**

4

**90**

6

# Iteration No. 4

Now nodes 1, 2, 3, 4 and 5 are connected. The remaining nodes are 6, 7 and 8. Among them, node 6 is at the shortest distance of 60 from node 3. So, connect node 3 to node 6 by a thick line. This is Iteration No. 5.

**60**

5

2

**70**

**60 60 80**

**100**

7

1

**40**

3

**120**

**50**

**80**

**50**

**60**

8

**30**

4

**90**

6

# Iteration No. 5

Now nodes 1, 2, 3, 4, 5 and 6 are connected. The remaining nodes are 7 and 8. Among them, node 8 is at the shortest distance of 30 from node 6. Consequently we connect node 6 to node 8 by a thick line. This is Iteration No. 6.

**60**

5

2

**70**

**60 60 80**

**100**

7

1

**40**

3

**120**

**50**

**80**

**50**

**60**

8

**30**

4

**90**

6

# Iteration No. 6

Now nodes 1, 2, 3, 4, 5, 6 and 8 are connected. The remaining node is 7. It is at the shortest distance of 50 from node 8. So, connect node 8 to node 7 by a thick line. This is Iteration No.7.

**60**

5

2

**70**

**60 60 80**

**100**

7

1

**40**

3

**120**

**50**

**80**

**50**

**60**

8

**30**

4

**90**

6

# Iteration No. 7

Now all the nodes 1, 2, 3, 4, 5, 6, 7 and 8 are connected by seven thick lines. Since no node is remaining, we have reached the stopping condition. Thus we obtain the following minimum spanning tree for the given network.

**60** 5

2

# 60

7

1 3

205

8

# 40 50

**50 60**

**30**

**Minimum Spanning Tree**

**QUESTIONS**

1. Explain the minimum spanning tree algorithm.
2. From the following network, find the minimum spanning tree.

# 100

**75**

6

2

**80**

**55**

**90**

1 3

**70**

**40**

**25**

**60**

5

4

**30**

1. Find the minimum spanning tree of the following network:

**12**

5

2

**15**

**5 8**

**2**

1 3

**10**

**13**

8

6

**9**

**4**

**10**

**5**

7

4

**4**

**PROJECT NETWORK**

# LESSON OUTLINE

* + The key concepts
  + Construction of project network diagram

# LEARNING OBJECTIVES

## After reading this lesson you should be able to

* understand the definitions of important terms
* understand the development of project network diagram
* work out numerical problems

# KEY CONCEPTS

Certain key concepts pertaining to a project network are described below:

# Activity

An activity means a work. A project consists of several activities. An activity takes time. It is represented by an arrow in a diagram of the network. For example, an activity in house construction can be flooring. This is represented as follows:

flooring

Construction of a house involves various activities. Flooring is an activity in this project. We can say that a project is completed only when all the activities in the project are completed.

# Event

It is the beginning or the end of an activity. Events are represented by circles in a project network diagram. The events in a network are called the **nodes**. **Example:**

# Start Stop

**Punching**

Starting a punching machine is an activity. Stopping the punching machine is another activity.

# Predecessor Event

The event just before another event is called the predecessor event.

# Successor Event

The event just following another event is called the successor event.

**Example:** Consider the following.

208

3

1

2

4

6

In this diagram, event 1 is predecessor for the event 2. Event 2 is successor to event 1.

Event 2 is predecessor for the events 3, 4 and 5. Event 4 is predecessor for the event 6.

Event 6 is successor to events 3, 4 and 5.

# Network

A network is a series of related activities and events which result in an end product or service. The activities shall follow a prescribed sequence. For example, while constructing a house, laying the foundation should take place before the construction of walls. Fitting water tapes will be done towards the completion of the construction. Such a sequence cannot be altered.

# Dummy Activity

A dummy activity is an activity which does not consume any time. Sometimes, it may be necessary to introduce a dummy activity in order to provide connectivity to a network or for the preservation of the logical sequence of the nodes and edges.

# Construction of a Project Network

A project network consists of a finite number of events and activities, by adhering to a certain specified sequence. There shall be a **start event** and an **end event (or stop event)**. All the other events shall be between the start and the end events. The activities shall be marked by directed arrows. An activity takes the project from one event to another event.

An event takes place at a point of time whereas an activity takes place from one point of time to another point of time.

# CONSTRUCTION OF PROJECT NETWORK DIAGRAMS

**Problem 1:**

Construct the network diagram for a project with the following activities**:**

|  |  |  |
| --- | --- | --- |
| Activity | Name of | Immediate |

|  |  |  |
| --- | --- | --- |
| EventEvent | Activity | Predecessor  Activity |
| 12 | A | - |
| 13 | B | - |
| 14 | C | - |
| 25 | D | A |
| 36 | E | B |
| 46 | F | C |
| 56 | G | D |

# Solution:

The start event is node 1.

The activities A, B, C start from node 1 and none of them has a predecessor activity. A joins nodes1 and 2; B joins nodes 1 and 3; C joins nodes 1 and 4. So we get the following:

**A**

2

1

**B**

3

**C**

4

This is a part of the network diagram that is being constructed.

Next, activity D has A as the predecessor activity. D joins nodes 2 and 5. So we get

1 **A** 2 **D** 5

Next, activity E has B as the predecessor activity. E joins nodes 3 and 6. So we get

1

**B**

3

**E**

6

Next, activity G has D as the predecessor activity. G joins nodes 5 and 6. Thus we obtain

**D**

**G**

2 5 6

Since activities E, F, G terminate in node 6, we get

5

**G**

3

**E**

6

4

# F

6 is the end event.

Combining all the pieces together, the following network diagram is obtained for the given project:

# D

2 5

# A G

**Start event End event**

1 **B** 3 **E** 6

# C F

4

We validate the diagram by checking with the given data.

# Problem 2:

Develop a network diagram for the project specified below:

|  |  |
| --- | --- |
| Activity | Immediate Predecessor Activity |
| A | **-** |
| B | A |
| C, D | B |
| E | C |
| F | D |
| G | E, F |

# Solution:

Activity A has no predecessor activity. i.e., It is the first activity. Let us suppose that activity A takes the project from event 1 to event 2. Then we have the following representation for A:

**A**

1

2

For activity B, the predecessor activity is A. Let us suppose that B joins nodes 2 and 3. Thus we get

1

**A**

2

**B**

3

Activities C and D have B as the predecessor activity. Therefore we obtain the following:

# C

**B**

4

2

3

**D**

5

Activity E has D as the predecessor activity. So we get

**C**

**E**

3

4

6

Activity F has D as the predecessor activity. So we get

3

**D**

5

**F**

6l

Activity G has E and F as predecessor activities. This is possible only if nodes 6 and 6l are one and the same. So, rename node 6l as node 6. Then we get

3

**D**

5

**F**

6!

and

4

**E**

6

**G**

7

5

# F

G is the last activity.

Putting all the pieces together, we obtain the following diagram the project network:

**Start event**

**C**

4

**E**

**End event**

1

**A**

2

**B**

3

**G**

6

7

**D**

**F**

5

The diagram is validated by referring to the given data.

**Note**: An important point may be observed for the above diagram. Consider the following parts in the diagram

**C**

**E**

3

4

6

and

3

**D**

5

**F**

6l

We took nodes 6 and 6l as one and the same. Instead, we can retain them as different nodes. Then, in order to provide connectivity to the network, we join nodes 6l and 6 by a dummy activity. Then we arrive at the following diagram for the project network:

4

**Start event**

**C**

**E**

6

1

**A**

2

**B**

3

**G**

**dummy activity**

**D**

5

**F**

6l

7

**End event**

**CRITICAL PATH METHOD (CPM)**

**LESSON OUTLINE**

* The concepts of critical path and critical activities
* Location of the critical path
* Evaluation of the project completion time

# LEARNING OBJECTIVES

## After reading this lesson you should be able to

* understand the definitions of critical path and critical activities
* identify critical path and critical activities
* determine the project completion time

# INTRODUCTION

The critical path method (CPM) aims at the determination of the time to complete a project and the important activities on which a manager shall focus attention.

# ASSUMPTION FOR CPM

In CPM, it is assumed that precise time estimate is available for each activity.

# PROJECT COMPLETION TIME

From the start event to the end event, the time required to complete all the activities of the project in the specified sequence is known as the project completion time.

# PATH IN A PROJECT

A continuous sequence, consisting of nodes and activities alternatively, beginning with the start event and stopping at the end event of a network is called a path in the network.

# CRITICAL PATH AND CRTICAL ACTIVITIES

Consider all the paths in a project, beginning with the start event and stopping at the end event. For each path, calculate the time of execution, by adding the time for the individual activities in that path.

The path with the largest time is called the **critical path** and the activities along this path are called the **critical activities** or **bottleneck activities**. The activities are called critical

because they cannot be delayed. However, a non-critical activity may be delayed to a certain extent. Any delay in a critical activity will delay the completion of the whole project. However, a certain permissible delay in a non –critical activity will not delay the completion of the whole project. It shall be noted that delay in a non-critical activity beyond a limit would certainly delay the completion the whole project. Sometimes, there may be several critical paths for a project. A project manager shall pay special attention to critical activities.

# Problem 1:

The following details are available regarding a project:

|  |  |  |
| --- | --- | --- |
| Activity | Predecessor Activity | Duration (Weeks) |
| A | **-** | 3 |
| B | A | 5 |
| C | A | 7 |
| D | B | 10 |
| E | C | 5 |
| F | D,E | 4 |

Determine the critical path, the critical activities and the project completion time.

# Solution:

First let us construct the network diagram for the given project. We mark the time estimates along the arrows representing the activities. We obtain the following diagram:

**Start event**

**B**

3

**D**

**End event**

**5 10**

1

**A 3**

2

**C**

5

**F 4**

6

**7**

**E**

4 **5**

Consider the paths, beginning with the start node and stopping with the end node. There are two such paths for the given project. They are as follows:

# Path I

**A**

**B**

**D**

**F**

1 2 3 5 6

**3 5 10 4**

with a time of 3 + 5 + 10 + 4 = 22 weeks.

# Path II

**A**

**C**

**E**

**F**

1 2 4 5 6

**3 7 5 4**

with a time of 3 + 7 + 5 + 4 = 19 weeks.

Compare the times for the two paths. Maximum of {22,19} = 22. We see that path I has the maximum time of 22 weeks. Therefore, path I is the critical path. The critical activities are A, B, D and F. The project completion time is 22 weeks.

We notice that C and E are non- critical activities. Time for path I - Time for path II = 22- 19 = 3 weeks.

Therefore, together the non- critical activities can be delayed upto a maximum of 3 weeks, without delaying the completion of the whole project.

# Problem 2:

Find out the completion time and the critical activities for the following project:

2

**D**

**20**

5

**A**

**8**

**G 8**

**B E H 11 K 6**

1

3

6

8

10

**10**

**16**

**I 14**

**L 5**

**C 7**

**J**

9

**F**

**25**

7

4

**10**

# Solution:

In all, we identify 4 paths, beginning with the start node of 1 and terminating at the end node of 10. They are as follows:

# Path I

**A**

**D**

**G**

**K**

1 2 5

8

10

**8 20 8 6**

Time for the path **=** 8 + 20 + 8 + 6 = 42 units of time.

# Path II

**B**

**E**

**H**

**K**

1 3 6

8

10

**10 16 11 6**

Time for the path **=** 10 + 16 + 11 + 6 = 43 units of time.

# Path III

**B**

**E**

**I**

**L**

1 3 6

9

10

**10 16 14 5**

Time for the path **=** 10 + 16 + 14 + 5 = 45 units of time.

# Path IV

**C**

**F**

**J**

**L**

1 4 7

9

10

**7 25 10 5**

Time for the path **=** 7 + 25 + 10 + 5 = 47 units of time.

Compare the times for the four paths. Maximum of {42, 43, 45, 47} = 47. We see that the following path has the maximum time and so it is the critical path:

**C**

**F**

**J**

**L**

1 4 7

9

10

**7 25 10 5**

The critical activities are C, F, J and L. The non-critical activities are A, B, D, E, G, H, I and

K. The project completion time is 47 units of time.

# Problem 3:

Draw the network diagram and determine the critical path for the following project:

|  |  |
| --- | --- |
| Activity | Time estimate (Weeks) |
| 1- 2 | 5 |
| 1- 3 | 6 |

|  |  |
| --- | --- |
| 1- 4 | 3 |
| 2 -5 | 5 |
| 3 -6 | 7 |
| 3 -7 | 10 |
| 4 -7 | 4 |
| 5 -8 | 2 |
| 6 -8 | 5 |
| 7 -9 | 6 |
| 8 -9 | 4 |

**Solution:** We have the following network diagram for the project:

2

**D 5**

5

**2 H**

**A**

**5**

1

**B 6**

3

**E 7**

6

**I 5**

8

**K**

**4**

9

**3 C**

**F**

**10**

**J**

7

**6**

4

**G**

**4**

# Solution:

We assert that there are 4 paths, beginning with the start node of 1 and terminating at the end node of 9. They are as follows:

# Path I

**A**

**D**

**H**

**K**

1 2 5

8

9

**5 5 2 4**

Time for the path **=** 5 + 5 + 2 + 4 = 16 weeks.

# Path II

**B**

**E**

**I**

**K**

1 3 6

8

9

**6 7 5 4**

Time for the path **=** 6 + 7 + 5 + 4 = 22 weeks.

# Path III

**B**

**F**

**J**

1 3 7 9

**6 10 6**

Time for the path **=** 6 + 10 + 6 = 16 weeks.

# Path IV

**C**

**G**

**J**

1 4 7 9

**3 4 6**

Time for the path **=** 3 + 4 + 6 = 13 weeks.

Compare the times for the four paths. Maximum of {16, 22, 16, 13} = 22. We see that the following path has the maximum time and so it is the critical path:

**B**

**E**

**I**

**K**

1 3 6

8

9

**6 7 5 4**

The critical activities are B, E, I and K. The non-critical activities are A, C, D, F, G, H and J.

The project completion time is 22 weeks.

# PERT

**LESSON OUTLINE**

* + The concept of PERT
  + Estimates of the time of an activity
  + Determination of critical path
  + Probability estimates

# LEARNING OBJECTIVES

## After reading this lesson you should be able to

* understand the importance of PERT
* locate the critical path
* determine the project completion time
* find out the probability of completion of a project before a stipulated time

# INTRODUCTION

Programme Evaluation and Review Technique (PERT) is a tool that would help a project manager in project planning and control. It would enable him in continuously monitoring a project and taking corrective measures wherever necessary. This technique involves statistical methods.

# ASSUMPTIONS FOR PERT

Note that in CPM, the assumption is that precise time estimate is available for each activity in a project. However, one finds most of the times that this is not practically possible.

In PERT, we assume that it is not possible to have precise time estimate for each activity and instead, probabilistic estimates of time alone are possible. A multiple time estimate approach is followed here. In probabilistic time estimate, the following 3 types of estimate are possible:

1. Pessimistic time estimate ( *tp* )
2. Optimistic time estimate ( *to* )
3. Most likely time estimate ( *tm* )

The optimistic estimate of time is based on the assumption that an activity will not involve any difficulty during execution and it can be completed within a short period. On the other hand, a pessimistic estimate is made on the assumption that there would be unexpected

problems during the execution of an activity and hence it would consume more time. The most likely time estimate is made in between the optimistic and the pessimistic estimates of time. Thus the three estimates of time have the relationship

*to*  *tm*  *tp* .

Practically speaking, neither the pessimistic nor the optimistic estimate may hold in reality and it is the most likely time estimate that is expected to prevail in almost all cases. Therefore, it is preferable to give more weight to the most likely time estimate.

We give a weight of 4 to most likely time estimate and a weight of 1 each to the pessimistic and optimistic time estimates. We arrive at a time estimate ( *te* ) as the weighted average of these estimates as follows:

*t*  *to*  4 *tm*  *tp e* 6

Since we have taken 6 units ( 1 for *tp* , 4 for *tm* and 1 for *to* ), we divide the sum by 6. With

this time estimate, we can determine the project completion time as applicable for CPM.

Since PERT involves the average of three estimates of time for each activity, this method is very practical and the results from PERT will be have a reasonable amount of reliability.

# MEASURE OF CERTAINTY

The 3 estimates of time are such that

*to*  *tm*  *tp* .

Therefore the range for the time estimate is *tp*  *to* .

The time taken by an activity in a project network follows a distribution with a standard deviation of one sixth of the range, approximately.

i.e., The standard deviation = **  *tp*  *to*

6

 *t*  *t* 2

and the variance = ** 2   *p o* 

 6 

The certainty of the time estimate of an activity can be analysed with the help of the variance. The greater the variance, the more uncertainty in the time estimate of an activity.

# Problem 1:

Two experts A and B examined an activity and arrived at the following time estimates.

Time Estimate

Expert

|  |  |  |  |
| --- | --- | --- | --- |
|  | *to* | *tm* | *tp* |
| A | 4 | 6 | 8 |
| B | 4 | 7 | 10 |

Determine which expert is more certain about his estimates of time:

# Solution:

 *t*  *t* 2

Variance ( ** 2 ) in time estimates =  *p o* 

 6 

 8  4 2 4

In the case of expert A, the variance =  6  9



 

 10  4 2

As regards expert B, the variance =  6  1



 

So, the variance is less in the case of A. Hence, it is concluded that the expert A is more certain about his estimates of time.

# Determination of Project Completion Time in PERT

**Problem 2:**

Find out the time required to complete the following project and the critical activities:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Activity | Predecessor  Activity | Optimistic time  estimate (to days) | Most likely time  estimate (tm days) | Pessimistic time  estimate (tp days) |
| A | - | 2 | 4 | 6 |
| B | A | 3 | 6 | 9 |
| C | A | 8 | 10 | 12 |
| D | B | 9 | 12 | 15 |
| E | C | 8 | 9 | 10 |
| F | D, E | 16 | 21 | 26 |
| G | D, E | 19 | 22 | 25 |
| H | F | 2 | 5 | 8 |
| I | G | 1 | 3 | 5 |

**Solution:**

From the three time estimates *tp* , *tm* and *to* , calculate *te* for each activity. We obtain the following table:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Activity | Optimistic time estimate (to) | 4 x Most likely time estimate | Pessimistic time estimate (tp) | to+ 4tm+ tp | Time estimate  *t*  *to*  4 *tm*  *tp e* 6 |
| A | 2 | 16 | 6 | 24 | 4 |
| B | 3 | 24 | 9 | 36 | 6 |
| C | 8 | 40 | 12 | 60 | 10 |
| D | 9 | 48 | 15 | 72 | 12 |
| E | 8 | 36 | 10 | 54 | 9 |
| F | 16 | 84 | 26 | 126 | 21 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| G | 19 | 88 | 25 | 132 | 22 |
| H | 2 | 20 | 8 | 30 | 5 |
| I | 1 | 12 | 5 | 18 | 3 |

Using the single time estimates of the activities, we get the following network diagram for the project.

**B** 3 **D F H** 6

**A**

1 **4** 2

**6 12 21 5**

**C 10 E** 5**G I** 8

**9 22 3**

4 7

Consider the paths, beginning with the start node and stopping with the end node. There are four such paths for the given project. They are as follows:

**Path I**

A B D F H

1 2 3

5 6 8

4 6 12 21 5

Time for the path: 4+6+12+21+5 = 48 days.

# Path II

A

B

D

G

I

1 2

3

12

5

7

8

4

6

6

3

Time for the path: 4+6+12+ 6+3 = 31 days.

# Path III

1

A 4

C

2 10

49

E

F 5

21

H 5

6

8

Time for the path: 4+10+9+ 21+5 = 49 days.

# Path IV

A

C

2

E 4

9

G

I

1

5

7

8

4

10

6

3

Time for the path: 4+10+9+ 6+3 = 32 days.

Compare the times for the four paths. Maximum of {48, 31, 49, 32} = 49.

We see that Path III has the maximum time.

Therefore the critical path is Path III. i.e., 1  2  4  5  6  8. The critical activities are A, C, E, F and H.

The non-critical activities are B, D, G and I. Project time (Also called project length) = 49 days.

**Problem 3:**

Find out the time, variance and standard deviation of the project with the following time estimates in weeks:

|  |  |  |  |
| --- | --- | --- | --- |
| Activity | Optimistic time  estimate (to) | Most likely time  estimate (tm) | Pessimistic time  estimate (tp) |
| 1-2 | 3 | 6 | 9 |
| 1-6 | 2 | 5 | 8 |
| 2-3 | 6 | 12 | 18 |
| 2-4 | 4 | 5 | 6 |
| 3-5 | 8 | 11 | 14 |
| 4-5 | 3 | 7 | 11 |
| 6-7 | 3 | 9 | 15 |
| 5-8 | 2 | 4 | 6 |
| 7-8 | 8 | 16 | 18 |

**Solution:**

From the three time estimates *tp* , *tm* and *to* , calculate *te* for each activity. We obtain the following table:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Activity | Optimistic time estimate (to) | 4 x Most likely time estimate | Pessimistic time estimate (tp) | to+ 4tm+ tp | Time estimate  *t*  *to*  4 *tm*  *tp e* 6 |
| 1-2 | 3 | 24 | 9 | 36 | 6 |
| 1-6 | 2 | 20 | 8 | 30 | 5 |
| 2-3 | 6 | 48 | 18 | 72 | 12 |
| 2-4 | 4 | 20 | 6 | 30 | 5 |
| 3-5 | 8 | 44 | 14 | 66 | 11 |
| 4-5 | 3 | 28 | 11 | 42 | 7 |
| 6-7 | 3 | 36 | 15 | 54 | 9 |
| 5-8 | 2 | 16 | 6 | 24 | 4 |
| 7-8 | 8 | 64 | 18 | 90 | 15 |

With the single time estimates of the activities, we get the following network diagram for the project.

C 3 F

12 11

D 5 G 5I A 6 7

2

4

4

1 5 B H

8

E 15

9

Consider the paths, begin6ning with the start node and stopping with the end node. There are three such paths for

the given project. They are as follows: 7

# Path I

A

C

1 2

F 3

I

5

8

6 12 11 4

Time for the path: 6+12+11+4 = 33 weeks.

# Path II

A D G I

1 2 4 5 8

6 5 7 4

Time for the path: 6+5+7+ 4= 22 weeks.

# Path III

B E H

1

5

6 9

175

8

Time for the path: 5+9+15 = 29 weeks.

Compare the times for the three paths. Maximum of {33, 22, 29} = 33.

It is noticed that Path I has the maximum time.

Therefore the critical path is Path I. i.e., 1  2  3  5  8 The critical activities are A, C, F and I.

The non-critical activities are B, D, G and H. Project time = 33 weeks.

**Calculation of Standard Deviation and Variance for the Critical Activities:**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Critical Activity | Optimistic time estimate  (to) | Most likely time estimate  (tm) | Pessimistic time estimate  (tp) | Range (tp - to) | Standard deviation =  **  *tp*  *to*  6 | Variance   *t*  *t* 2  ** 2   *p o*    6  |
| A: 12 | 3 | 6 | 9 | 6 | 1 | 1 |
| C: 23 | 6 | 12 | 18 | 12 | 2 | 4 |
| F: 35 | 8 | 11 | 14 | 6 | 1 | 1 |
| I: 58 | 2 | 4 | 6 | 4 | 2/3 | 4/9 |

Variance of project time (Also called Variance of project length) =

Sum of the variances for the critical activities = 1+4+1+ 4/9 = 58/9 Weeks. Standard deviation of project time = √Variance = √58/9 = 2.54 weeks.

**Problem 4**

A project consists of seven activities with the following time estimates. Find the probability that the project will be completed in 30 weeks or less.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Activity | Predecessor Activity | Optimistic time estimate (to days) | Most likely time estimate (tm days) | Pessimistic time estimate (tp days) |
| A | - | 2 | 5 | 8 |
| B | A | 2 | 3 | 4 |
| C | A | 6 | 8 | 10 |
| D | A | 2 | 4 | 6 |
| E | B | 2 | 6 | 10 |
| F | C | 6 | 7 | 8 |
| G | D, E, F | 6 | 8 | 10 |

**Solution:**

From the three time estimates *tp* , *tm* and *to* , calculate *te*

following table:

for each activity. The results are furnished in the

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Activity | Optimistic time estimate (to) | 4 x Most likely time estimate | Pessimistic time estimate (tp) | to+ 4tm+ tp | Time estimate  *t*  *to*  4 *tm*  *tp e* 6 |
| A | 2 | 20 | 8 | 30 | 5 |
| B | 2 | 12 | 4 | 18 | 3 |
| C | 6 | 32 | 10 | 48 | 8 |
| D | 2 | 16 | 6 | 24 | 4 |
| E | 2 | 24 | 10 | 36 | 6 |
| F | 6 | 28 | 8 | 42 | 7 |
| G | 6 | 32 | 10 | 48 | 8 |

With the single time estimates of the activities, the following network diagram is constructed for the project.

3

B 3

6 E

C

8

4F

7

A 5

D 4

1 2

G 8

5

6

Consider the paths, beginning with the start node and stopping with the end node. There are three such paths for

the given project. They are as follows:

# Path I

A

B

1 2

5 3

E 3

6

G

5

6

8

Time for the path: 5+3+6+8 = 22 weeks.

# Path II

A C F G

1 2 4 5 6

5 8 7 8

Time for the path: 5+8+7+ 8 = 28 weeks.

# Path III

A

D

1 2

G 5

8

6

5

4

Time for the path: 5+4+8 = 17 weeks.

Compare the times for the three paths. Maximum of {22, 28, 17} = 28.

It is noticed that Path II has the maximum time.

Therefore the critical path is Path II. i.e., 1  2  4  5  6. The critical activities are A, C, F and G.

The non-critical activities are B, D and E. Project time = 28 weeks.

**Calculation of Standard Deviation and Variance for the Critical Activities:**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Critical Activity | Optimistic time estimate (to) | Most likely time estimate (tm) | Pessimistic time estimate (tp) | Range (tp -to) | Standard deviation =  **  *tp*  *to*  6 | Variance   *t*  *t* 2  ** 2   *p o*    6  |
| A: 12 | 2 | 5 | 8 | 6 | 1 | 1 |
| C: 24 | 6 | 8 | 10 |  | 2 | 4 |
|  |  |  |  | 4 | 3 | 9 |
| F: 45 | 6 | 7 | 8 |  | 1 | 1 |
|  |  |  |  | 2 | 3 | 9 |
| G: 56 | 6 | 8 | 10 |  | 2 | 4 |
|  |  |  |  | 4 | 3 | 9 |

Standard deviation of the critical path = √2 = 1.414 The standard normal variate is given by the formula

*Z*  *Given value of t*

* *Expected value of t* in *the critical path*

*SD for the critical path*

30  28

So we get

*Z*  = 1.414

1.414

We refer to the Normal Probability Distribution Table. Corresponding to Z = 1.414, we obtain the value of 0.4207 We get 0.5 + 0.4207 = 0. 9207

Therefore the required probability is 0.92

i.e., There is 92% chance that the project will be completed before 30 weeks. In other words, the chance that it will be delayed beyond 30 weeks is 8%