**UNIT 4 ( Inventory Theory)**

The word ‘inventory’ means simply a stock of idle resources of any kind having an economic value. In other words, inventory means a physical stock of goods, which is kept in hand for smooth and eﬃcient running of future aﬀairs of an organization. It may consist of raw materials, work-in-progress, spare parts/consumables, finished goods, human resources such as unutilized labor, financial resources such as working capital, etc. It is not necessary that an organization has all these inventory classes but whatever may be the inventory items, they need eﬃcient management as generally a substantial amount of money is invested in them. The basic inventory decisions in- clude: 1) *How much to order?* 2) *When to order?* 3) *How much safety stock should be kept?* The problems faced by diﬀerent organizations have necessitated the use of scientific techniques in the management of inventories known as inventory control. Inventory control is concerned with the acquisition, storage, and handling of inventories so that the inventory is available whenever needed and the associated total cost is minimized.

# Reasons for Carrying Inventory

1.2

Inventories are carried by organisations because of the following major reasons :

1. Improve customer service- An inventory policy is designed to respond to indi- vidual customer’s or organization’s request for products and services.

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1. Reduce costs- Inventory holding or carrying costs are the expenses that are in- curred for storage of items. However, holding inventory items in the warehouse can indirectly reduce operating costs such as loss of goodwill and/or loss of po- tential sale due to shortage of items. It may also encourage economies of pro- duction by allowing larger, longer and more production runs.
2. Maintenance of operational capability- Inventories of raw materials and work- in-progress items act as buﬀer between successive production stages so that downtime in one stage does not aﬀect the entire production process.
3. Irregular supply and demand- Inventories provide protection against irregular supply and demand; an unexpected change in production and delivery sched- ule of a product or a service can adversely aﬀect operating costs and customer service level.
4. Quantity discount- Large size orders help to take advantage of price-quantity discount. However, such an advantage must keep a balance between the storage cost and costs due to obsolescence, damage, theft, insurance, etc.
5. Avoiding stockouts (shortages)- Under situations like, labor strikes, natural disasters, variations in demand and delays in supplies, etc., inventories act as buﬀer against stock out as well as loss of goodwill.

# Costs Associated with Inventories

1.3

Various costs associated with inventory control are often classified as follows :

1. *Purchase (or production) cost*: It is the cost at which an item is purchased, or if an item is produced.
2. *Carrying (or holding) cost*: The cost associated with maintaining inventory is known as holding cost. It is directly proportional to the quantity kept in stock and the time for which an item is held in stock. It includes handling cost, main- tenance cost, depreciation, insurance, warehouse rent, taxes, etc.
3. *Shortage (or stock out) cost*: It is the cost which arises due to running out of stock. It includes the cost of production stoppage, loss of goodwill, loss of profitability, special orders at higher price, overtime/idle time payments, loss of opportunity to sell, etc.
4. *Ordering (or set up) cost*: The cost incurred in replenishing the inventory is known as ordering cost. It includes all the costs relating to administration (such as salaries of the persons working for purchasing, telephone calls, computer costs, postage, etc.), transportation, receiving and inspection of goods, processing pay- ments, etc. If a firm produces its own goods instead of purchasing the same from an outside source, then it is the cost of resetting the equipment for production.

# Basic Terminologies

1.4

The followings are some basic terminologies which are used in inventory theory:

1. Demand

It is an eﬀective desire which is related to particular time, price, and quantity. The demand pattern of a commodity may be either deterministic or probabilistic. In case of deterministic demand, the quantities needed in future are known with certainty. This can be fixed (static) or can vary (dynamic) from time to time. On the contrary, probabilistic demand is uncertain over a certain period of time but its pattern can be described by a known probability distribution.

1. Ordering cycle

An ordering cycle is defined as the time period between two successive replen- ishments. The order may be placed on the basis of the following two types of inventory review systems:

* + *Continuous review*: In this case, the inventory level is monitored continu- ously until a specified point (known as reorder point) is reached. At this point, a new order is placed.
	+ *Periodic review*: In this case, the orders are placed at equally spaced intervals of time. The quantity ordered each time depends on the available inventory level at the time of review.
1. Planning period

This is also known as time horizon over which the inventory level is to be con- trolled. This can be finite or infinite depending on the nature of demand.

1. Lead time or delivery lag

The time gap between the moment of placing an order and actually receiving it is referred to as lead time. Lead time can be deterministic (constant or variable) or probabilistic.

1. Buﬀer (or safety) stock

Normally, demand and lead time are uncertain and cannot be predetermined completely. So, to absorb the variation in demand and supply, some extra stock is kept. This extra stock is known as buﬀer stock.

1. Re-order level

The level between maximum and minimum stocks at which purchasing activity must start for replenishment is known as re-order level.

# Economic Order Quantity (EOQ)

1.5

The concept of economic ordering quantity (EOQ) was first developed by F. Harris in 1916. The concept is as follows: Management of inventory is confronted with a set of opposing costs. As the lot size increases, the carrying cost increases while the ordering cost decreases. On the other hand, as the lot size decreases, the carrying cost decreases but the ordering cost increases. The two opposite costs can be shown graphically by plotting them against the order size as shown in Fig. 1.1 below :

Fig. 1.1: Graph of EOQ

Economic ordering quantity(EOQ) is that size of order which minimizes the average total cost of carrying inventory and ordering under the assumed conditions of cer- tainty and the total demand during a given period of time is known.

# List of Symbols

1.6

The following symbols are used in connection with the inventory models presented in this chapter :

|  |  |  |
| --- | --- | --- |
| *c* | = | purchase (or manufacturing) cost of an item |
| *c*1 | = | holding cost per quantity unit per unit time |
| *c*2 | = | shortage cost per quantity unit per unit item |
| *c*3 | = | ordering (set up) cost per order (set up) |
| *R* | = | demand rate |
| *P* | = | production rate |
| *t* | = | scheduling period which is variable |
| *tp* | = | prescribe scheduling period |
| *D* | = | total demand or annual demand |
| *q* | = | lot (order) size |
| *L* | = | lead time |
| *x* | = | random demand |
| *f* (*x*) | = | probability density function for demand *x*. |
| *z* | = | order level |

# Deterministic Inventory Models

1.7

## Model I(a): EOQ model without shortage

The basic assumptions of the model are as follows:

* + - * Demand rate *R* is known and uniform.
			* Lead time is zero or a known constant.
			* Replenishment rate is infinite, i.e., replenishments are instantaneous.
			* Shortages are not permitted.
			* Inventory holding cost is *c*1 per unit per unit time.
			* Ordering cost is *c*3 per order.

Our objective is to determine the economic order quantity *q*∗ which minimizes the average total cost of the inventory system. An inventory-time diagram with inventory level on the vertical axis and time on the horizontal axis is shown in Fig. 1.2. Since the actual consumption of inventory varies constantly, the concept of average inventory is applicable here. *Average Inventory* = 1*/*2[*maximum level* + *minimum level*] = (*q* + 0)*/*2 = *q/*2*.*

Fig. 1.2: Inventory-time diagram when lead time is a known constant

Thus, the average inventory carrying cost is = *average inventory* × *holding cost* = 1 *qc*1*.*

2

The average ordering cost is (*R/q*)*c*3. Therefore, the average total cost of the inventory system is given by

*C*(*q*) = 1 *c*1*q* + *c*3*R.* (1.1)

2 *q*

Since the minimum average total cost occurs at a point when average ordering cost

and average inventory carrying cost are equal, therefore, we have 1 *c*1*q* = *c*3*R*

which

2 *q*

gives the optimal order quantity

*q*∗ = 2*c*3*R.* (1.2)

√

*c*1

This result was derived independently by F.W. Harris and R.H. Wilson in the year 1915. Thats why the model is called *Harris-Wilson model*.

Characteristics of Model I(a)

1. Optimal ordering interval *t*∗ = *q*∗*/R* = √ 2*c*3

*c*1*R*

1. Minimum average total cost *C*min = *C*(*q*∗) = √2*c*1*c*3*R*

If in Model I(a), the ordering cost is taken as (*c*3 + *kq*) where *k* is the ordering cost per unit item ordered then there will be no change in the optimal order quantity *q*∗.

In this case, the average total cost is

*C*(*q*) = 1 *c*1*q* + *c*3*R* + *kR.* (1.3)

2 *q*

Example 1.1: A manufacturing company purchases 9000 parts of a machine for its annual requirements, ordering one month usage at a time. Each part costs Rs.20.

The ordering cost per order is Rs. 15 and the carrying charges are 15% of the average inventory per year. Suggest a more economic purchasing policy for the company. How much would it save the company per year ?

Solution: Given that *R* = 9000 parts/year, *c*1 = (15*/*100)×20 = *Rs.*3 per part/year, *c*3 =

*Rs.*15 per order. Using Harris-Wilson formula,

√

*q*∗ = 2*c*3*R/c*1 = 300 *units*

*t*∗ = *q*∗*/R* = 1*/*30 *year* = 12 *days* (*approx.*)

*Cmin* = √2*c*1*c*3*R* = *Rs.*900

If the company follows the policy of ordering every month, then lot size of inventory each month *q* = 9000*/*12 = 750 *parts*, annual storage cost = *c*1(*q/*2) = Rs. 1125,

annual ordering cost = 15 × 12 = Rs. 180.

The total cost per year = 1125 + 180 = Rs. 1305.

Therefore, the company should purchase 300 parts at time interval of 12 days instead of ordering 750 parts each month. Then there will be a net saving of Rs. 405.

Lot Size in Discrete Units

Let the lot size *q* be constrained to values *u,* 2*u,* 3*u,* · · · Then the necessary conditions for optimal *q*, i.e., *q*∗ are

*C*(*q*∗) ≤ *C*(*q*∗ + *u*) (1.4)

*C*(*q*∗) ≤ *C*(*q*∗ − *u*) (1.5)

From equations (1.4) and (1.5), we get by simplifying

*q*∗(*q*∗ − *u*) ≤ 2*Rc*3 ≤ *q*∗(*q*∗ + *u*) (1.6)

*c*1

Example 1.2: Demand in an inventory system is at a constant and uniform rate of 2400 kg. per year. The carrying cost is Rs. 5 per kg. per year. No shortage is allowed. The replenishment cost is Rs. 22 per order. The lot size can only be in 100 kg. unit. What is the optimal lot size of the system ?

Solution: Given that *R* = 2400*kg/year, c*1 = *Rs.*5*/kg/year, c*3 = *Rs.*22 *per order, u* = 100 *kg.* The optimal lot size *q*∗ has to satisfy the inequality

*q*∗(*q*∗ − *u*) ≤ 2*c*3*R* ≤ *q*∗(*q*∗ + *u*)

*c*1

*i.e., q*∗(*q*∗ − 100) ≤ 21120 ≤ *q*∗(*q*∗ + 100) By trial and error, we find the optimal lot size *q*∗ = 200 kg.

Sensitivity of Lot Size System

The average cost *C*(*q*) of the lot size system is a function of the controllable variable

√

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*q*

*c*1

*q* where *C*(*q*) = 1 *c*1*q*+ *c*3*R* . The optimal results are *q*∗ =  2*c*3*R* and *C*∗ = *C*(*q*∗) = √2*c*1*c*3*R*.

Suppose that instead of the optimal lot size *q*∗, the decision maker uses another lot size *q*′ which is related to *q*∗ by the relation *q*′ = *bq*∗*, b >* 0.

Let *C*′ designate the average total cost of the system then. We use the ratio *C*′ */C*∗

as a measure of sensitivity. It can be shown that *C*′ */C*∗ = (1 + *b*2)*/*(2*b*).

So, the measure of sensitivity is a function of *b* and is independent of the other pa- rameters *c*1*, c*3 and *R*.

## Model I(b): EOQ Model with Diﬀerent Rates of Demand

This inventory system operates on the assumptions of Model I(a) except that the de- mand rates are diﬀerent in diﬀerent cycles but order quantity is fixed in each cycle. The objective is to determine the order size in each reorder cycle that will minimize the total inventory cost. Suppose that the total demand *D* is specified over the plan- ning period *T* . If *t*1*, t*2*, ..., tn* denote the lengths of successive *n* inventory cycles and *D*1*, D*2*, ..., Dn* are the demand rates in these cycles, respectively, then the total period *T* is given by *T* = *t*1+*t*2+*...*+*tn*. Fig. 1.3 depicts the inventory system under consideration.

Fig. 1.3: Inventory-time diagram for diﬀerent cycles

Suppose that each time a fixed quantity *q* is ordered. Then the number of orders in

the time period *T* is *n* = *D/q*. Thus, the inventory carrying for the time period *T* is

1 1 1 1 1

2 *qt*1*c*1 + 2 *qt*2*c*1 + *...* + 2 *qtnc*1 = 2 *qc*1(*t*1 + *t*2 + *...* + *tn*) = 2 *qc*1*T*

Total ordering cost = (Number of orders) × *c*3 = *D c*3

*q*

Hence, the total inventory cost is *C*(*q*) = 1 *c*1*qT* + *c*3*D*

2 *q*

The optimal ordering quantity (*q*∗) is then determined by the first order condition as

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*q*∗ = 2*c*3(*D/T* )

*c*1

The minimum total inventory cost is obtained by substituting the value of *q*∗ in the cost equation, i.e.,

*C*min = √2*c*1*c*3(*D/T* )

Here we observe from the optimal results that the fixed demand rate *R* in Model I(a) is replaced by the average demand rate (*D/T* ) in this model.