**UNIT – 1**

 **SETS**

**SET**: A collection of well defined distict obejects is called set.

For example: the set of all living human beings, the set of all cities in India, the set of all sentences of some language, the set of all prime numbers, and so on. Each living human being is an element of the set of all living human beings.

Notation of a Set:

A set is usually denoted by capital letters and elements are denoted by small letters

If x is an element of set A, then we say x ϵ A. **[x belongs to A]**

If x is not an element of set A, then we say x ∉ A. **[x does not belong to A]**

**For example:**

The collection of vowels in the English alphabet.

**Solution :**

Let us denote the set by V, then the elements of the set are a, e, i, o, u or we can say, V = [a, e, i, o, u].

We say a ∈ V, e ∈ V, i ∈ V, o ∈ V and u ∈ V.

Also, we can say b ∉ V, c ∉ v, d ∉ v, etc.

In principle, any [finite set](https://www.britannica.com/science/finite-set) can be defined by an explicit list of its members, but specifying infinite sets requires a rule or pattern to indicate membership; for example, the ellipsis in { 1, 2, 3, 4, 5, 6, 7, …} indicates that the list of [natural numbers](https://www.britannica.com/science/natural-number) ℕ goes on forever.

**Null Set** : The empty (or void, or [null](https://www.britannica.com/science/null-set)) set, symbolized by {} or Ø, contains no elements at all. Nonetheless, it has the status of being a set.

**Subset** :

A set A is called a subset of a set B (symbolized by A ⊆ B) if all the members of A are also members of B. If A and B are two sets, and every element of set A is also an element of set B, then A is called a subset of B and we write it as **A ⊆ B** or **B ⊇ A**

The symbol **⊂** stands for ‘is a subset of’ or ‘is contained in’ • Every set is a subset of itself, i.e., A ⊂ A, B ⊂ B. • Empty set is a subset of every set. • Symbol ‘⊆’ is used to denote ‘is a subset of’ or ‘is contained in’. • A ⊆ B means A is a subset of B or A is contained in B. • B ⊆ A means B contains A.

**For example;**

**1.** Let A = {2, 4, 6}

B = {6, 4, 8, 2}

Here A is a subset of B

Since, all the elements of set A are contained in set B.

But B is not the subset of A

Since, all the elements of set B are not contained in set A.

2. Let A = {1, 2, 3, 4}

B = {4, 5, 6, 7}

Here A ⊄ B and also B ⊄ C

[**⊄** denotes ‘not a subset of’]

**Super Set:**

Whenever a set A is a subset of set B, we say the B is a superset of A and we write, B ⊇ A.

Symbol ⊇ is used to denote ‘is a super set of’

**For example;**

A = {a, e, i, o, u}

B = {a, b, c, ............., z}

Here A ⊆ B i.e., A is a subset of B but B ⊇ A i.e., B is a super set of A

**Proper Subset:**

If A and B are two sets, then A is called the proper subset of B if A ⊆ B but B ⊇ A i.e., A ≠ B. The symbol ‘⊂’ is used to denote proper subset. Symbolically, we write A ⊂ B.

**For example;**

**1.** A = {1, 2, 3, 4}

Here n(A) = 4

B = {1, 2, 3, 4, 5}

Here n(B) = 5

We observe that, all the elements of A are present in B but the element ‘5’ of B is not present in A.

So, we say that A is a proper subset of B.
Symbolically, we write it as A ⊂ B

**Notes:**

No set is a proper subset of itself.

Null set or ∅ is a proper subset of every set

**Power Set.** For every set xthere is a set P(x) whose elements are all the subsets of x.

**For example;**

If A = {p, q} then all the subsets of A will be

P(A) = {∅, {p}, {q}, {p, q}}

Number of elements of P(A) = n[P(A)] = 4 = 22

In general, n[P(A)] = 2m where m is the number of elements in set A.

**Operations on sets**

**UNION OF SETS**

The symbol ∪ is employed to denote the [union](https://www.britannica.com/science/union-set-theory) of two sets. Thus, the set *A* ∪ *B*—read “*A* union *B*” or “the union of *A* and *B*”—is defined as the set that consists of all elements belonging to either set *A* or set *B* (or both). The symbol for denoting union of sets is ‘**∪**’.

**For example;**

Let set A = {2, 4, 5, 6}
and set B = {4, 6, 7, 8}

Taking every element of both the sets A and B, without repeating any element, we get a new set = {2, 4, 5, 6, 7, 8}

This new set contains all the elements of set A and all the elements of set B with no repetition of elements and is named as **union of set A and B**.

The symbol used for the union of two sets is ‘**∪**’.

Therefore, symbolically, we write union of the two sets A and B is A ∪ B which means A union B.

Therefore, A ∪ B = {x : x ∈ A or x ∈ B}

**Notes:**

A and B are the subsets of A ∪ B

The union of sets is commutative, i.e., A ∪ B = B ∪ A.

The operations are performed when the sets are expressed in roster form.

Some properties of the operation of union:

(i) A∪B = B∪A                      **(Commutative law)**

(ii) A∪(B∪C) = (A∪B)∪C         **(Associative law)**

(iii) A ∪ ϕ = A                      **(Law of identity element, is the identity of ∪)**

(iv) A∪A = A                        **(Idempotent law)**

(v) U∪A = U                        **(Law of ∪)** ∪ is the universal set.

**Notes:**

A ∪ ϕ = ϕ ∪ A = A i.e. union of any set with the empty set is always the set itself.

**INTERSECTION OF SETS**

The [intersection](https://www.britannica.com/science/intersection-set-theory) operation is denoted by the symbol ∩. The set *A* ∩ *B*—read “*A* intersection *B*” or “the intersection of *A* and *B*”—is defined as the set composed of all elements that belong to both *A* and *B*.

The symbol for denoting intersection of sets is ‘**∩**‘.

**For example:**

Let set A = {2, 3, 4, 5, 6}

and set B = {3, 5, 7, 9}

In this two sets, the elements 3 and 5 are common. The set containing these common elements i.e., {3, 5} is the intersection of set A and B.

The symbol used for the intersection of two sets is ‘**∩**‘.

Therefore, symbolically, we write intersection of the two sets A and B is A ∩ B which means A intersection B.

The intersection of two sets A and B is represented as A ∩ B = {x : x ∈ A and x ∈ B}

examples to find intersection of two given sets:

**1.** If A = {2, 4, 6, 8, 10} and **B** = {1, 3, 8, 4, 6}. Find intersection of two set A and B.

**Solution:**

A ∩ B = {4, 6, 8}

Therefore, 4, 6 and 8 are the common elements in both the sets.

**2.** If X = {a, b, c} and **Y** = {ф}. Find intersection of two given sets X and Y.

**Solution:**

X ∩ Y = { }

**3.** If set A = {4, 6, 8, 10, 12}, set B = {3, 6, 9, 12, 15, 18} and set C = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

(i) Find the intersection of sets A and B.

(ii) Find the intersection of two set B and C.

(iii) Find the intersection of the given sets A and C.

**Solution:**

(i) Intersection of sets A and B is A ∩ B

Set of all the elements which are common to both set A and set B is {6, 12}.

(ii) Intersection of two set B and C is B ∩ C

Set of all the elements which are common to both set B and set C is {3, 6, 9}.

(iii) Intersection of the given sets A and C is A ∩ C

Set of all the elements which are common to both set A and set C is {4, 6, 8, 10}.

**Notes:**

A ∩ B is a subset of A and B.

Intersection of a set is commutative, i.e., A ∩ B = B ∩ A.

Operations are performed when the set is expressed in the roster form.

Some properties of the operation of intersection

(i) A∩B = B∩A (Commutative law)

(ii) (A∩B)∩C = A∩ (B∩C) (Associative law)

(iii) ϕ ∩ A = ϕ (Law of ϕ)

(iv) U∩A = A (Law of ∪)

(v) A∩A = A (Idempotent law)

(vi) A∩(B∪C) = (A∩B) ∪ (A∩C) (Distributive law) Here ∩ distributes over ∪

Also, A∪(B∩C) = (AUB) ∩ (AUC) (Distributive law) Here ∪ distributes over ∩

**Notes:**

A ∩ ϕ = ϕ ∩ A = ϕ i.e. intersection of any set with the empty set is always the empty set.

**DIFFERENCE OF SETS**

If A and B are two sets, then their difference is given by A - B or B - A.

• If A = {2, 3, 4} and B = {4, 5, 6}

A - B means elements of A which are not the elements of B.

i.e., in the above example A - B = {2, 3}

In general, B - A = {x : x ∈ B, and x ∉ A}

• If A and B are disjoint sets, then A – B = A and B – A = B

Ex.

**1.** A = {1, 2, 3} and B = {4, 5, 6}. The two sets are disjoint as they do not have any elements in common.

(i) A - B = {1, 2, 3} = A

(ii) B - A = {4, 5, 6} = B

**UNIVERSAL SET**

A set which contains all the elements of other given sets is called a **universal set**. The symbol for denoting a universal set is **∪** or **ξ**.

**For example;**

**1.** If A = {1, 2, 3}      B = {2, 3, 4}      C = {3, 5, 7}

then U = {1, 2, 3, 4, 5, 7}

[Here A ⊆ U, B ⊆ U, C ⊆ U and U ⊇ A, U ⊇ B, U ⊇ C]

**2.** If P is a set of all whole numbers and Q is a set of all negative numbers then the universal set is a set of all integers.

**3.** If A = {a, b, c}      B = {d, e}      C = {f, g, h, i}

then U = {a, b, c, d, e, f, g, h, i} can be taken as universal set.

**COMPLEMENT OF A SET** :

 For any subset *A* of *U*, the [complement](https://www.britannica.com/science/complement-set-theory) of *A* (symbolized by *A*′ or *U* − *A*) is defined as the set of all elements in the universe *U* that are not in *A*.

 For example, if the universe consists of the 26 letters of the alphabet, the complement of the set of vowels is the set of consonants.

Cardinal Number:

The number of distinct elements in a finite set is called its cardinal number. It is denoted as n(A) and read as ‘the number of elements of the set’.

**For example:**

(i) Set A = {2, 4, 5, 9, 15} has 5 elements.

Therefore, the cardinal number of set A = 5. So, it is denoted as n(A) = 5.

(ii) Set B = {w, x, y, z} has 4 elements.

Therefore, the cardinal number of set B = 4. So, it is denoted as n(B) = 4.

(iii) Set C = {Florida, New York, California} has 3 elements.

Therefore, the cardinal number of set C = 3. So, it is denoted as n(C) = 3.

Examples:

1. Let A and B be two finite sets such that n(A) = 20, n(B) = 28 and n(A ∪ B) = 36, find n(A ∩ B).

**Solution:**

Using the formula n(A ∪ B) = n(A) + n(B) - n(A ∩ B).

then n(A ∩ B) = n(A) + n(B) - n(A ∪ B)

                     = 20 + 28 - 36

                     = 48 - 36

                     = 12

**2.** If n(A - B) = 18, n(A ∪ B) = 70 and n(A ∩ B) = 25, then find n(B).

**Solution:**

Using the formula n(A∪B) = n(A - B) + n(A ∩ B) + n(B - A)

                                 70 = 18 + 25 + n(B - A)

                                 70 = 43 + n(B - A)

                         n(B - A) = 70 - 43

                         n(B - A) = 27

Now n(B) = n(A ∩ B) + n(B - A)

               = 25 + 27

               = 52

**3.** In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both coffee and tea?

**Solution:**

Let A = Set of people who like cold drinks.

     B = Set of people who like hot drinks.

*Given*

(A ∪ B) = 60            n(A) = 27       n(B) = 42 then;

n(A ∩ B) = n(A) + n(B) - n(A ∪ B)

            = 27 + 42 - 60

            = 69 - 60 = 9

            = 9

Therefore, 9 people like both tea and coffee.

CARTESIAN PRODUCT

In [analytic geometry](https://www.britannica.com/science/analytic-geometry), the points on a Cartesian grid are ordered pairs (*x*, *y*) of numbers. In general, (*x*, *y*) ≠ (*y*, *x*); ordered pairs are defined so that (*a*, *b*) = (*c*, *d*) if and only if both *a* = *c* and *b* = *d*. In contrast, the set {*x*, *y*} is identical to the set {*y*, *x*} because they have exactly the same members.

The Cartesian product of two sets *A* and *B*, denoted by *A* × *B*, is defined as the set consisting of all ordered pairs (*a*, *b*) for which *a* ∊ *A* and *b* ∊ *B*. For example, if *A* = {*x*, *y*} and *B* = {3, 6, 9}, then *A* × *B* = {(*x*, 3), (*x*, 6), (*x*, 9), (*y*, 3), (*y*, 6), (*y*, 9)}.