

①

If  $u = \sin^{-1}(x/y) + \tan^{-1}(y/x)$ , then find the value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ .

②

$$\text{Evaluate } \int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dxdy}{1+x^2+y^2}$$

③ Give a brief description of various Set Theoretical Operations and law of Set Theory.

④ If  $R$  be a relation in the set of integer  $I$  defined by  $R = \{(x,y) : x \in I, y \in I, x-y = 5k \text{ or } x-y \text{ is divisible by } 5\}$ .

Prove that  $R$  is an equivalence relation.

⑤ Give an example of relation which is :

- (a) Reflexive and transitive but not symmetric.
- (b) Symmetric and transitive but not reflexive.
- (c) Reflexive and symmetric but not transitive.
- (d) Reflexive and transitive but neither symmetric nor antisymmetric

⑥ Let  $A = \{1, 2, 3, 4\}$  and  $R = \{(1,2), (2,3), (3,4), (2,4)\}$ . Find transitive closure of  $R$ .

⑦ Prove that if  $R$  is an equivalence relation on set  $A$  then  $R^{-1}$  is also an equivalence relation on  $A$ .

⑥ If  $f(x) = \log\left(\frac{1+x}{1-x}\right)$ , show that

$$f(x) + f(y) = f\left(\frac{x+y}{1+xy}\right)$$

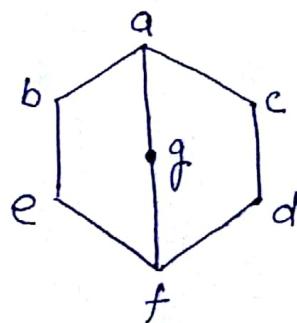
⑦ Consider the set  $N \times N$ , the set of ordered pairs of natural numbers. Let  $R$  be a relation in  $N \times N$  which is defined by  $(a,b) R (c,d)$  iff  $ad = bc$ .

Prove that  $R$  is an equivalence relation.

⑧ Define distributive lattice and prove that in a distributive lattice, if an element has a complement then this complement is unique.

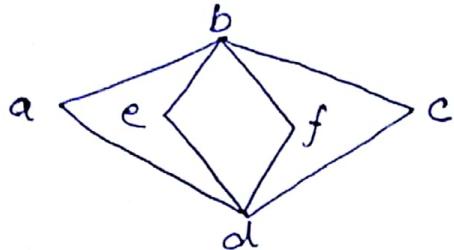
⑨ Draw the Hasse diagram of  $(A, \leq)$ , where  $A = \{3, 4, 12, 24, 48, 72\}$  and relation  $\leq$  be such that  $a \leq b$  if  $a$  divides  $b$ .

⑩ Determine whether the given Hasse diagram represents a complement lattice or not?



⑪ Let  $D(81)$  be the set of all positive divisors of 81, then show that  $D(81)$  under the binary relation 'divides' is a poset. Is the poset totally ordered?

- (14) Show that dual of a lattice is a lattice.
- (15) Show that the lattice  $L$  represented by diagram is complemented but not distributive



- (16) If  $u = \tan^{-1} \left( \frac{x^3 + y^3}{x + y} \right)$ , show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$$

- (17) If  $z = e^u f(v)$   
 $u = ax + by$  &  $v = ax - by$ .

$$\text{Show that: } b \frac{\partial z}{\partial x} + a \frac{\partial z}{\partial y} = 2abz.$$

- (18) State and prove Euler's Theorem.

- (19) If  $u = e^{xyz}$ ; show that:

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2) e^{xyz}$$

- (20) Discuss that maxima and minima of  $u = xy(1-x-y)$

- (21) Show that the lines whose directions cosines are given by the equations  $2l + 2m - n = 0$  and  $mn + nl + lm = 0$  are at right angle.

- (22) Find the equation to the sphere through the circle:  
 $x^2 + y^2 + z^2 = 9$ ,  $2x + 3y + 4z = 5$   
and the point  $(1, 2, 3)$ .

- (23) Prove that the lines  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and  
 $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$  are coplanar. Find their  
point of intersection. Also find the equation  
of the plane in which they lie.

- (24) Find the equation of the plane through the line  
of intersection of the planes  $ax+by+cz+d=0$   
and  $\alpha x+\beta y+\gamma z+\delta=0$  parallel to  $x$ -axis.

- (25) The projection of a line on axis are  $5, 10, 10$ . Find  
the length and direction cosines.

- (26) Evaluate  $\int_{x=0}^2 \int_{y=0}^3 \int_{z=0}^1 (x+y+z) dx dy dz$ .

- (27) Find the area between the parabola  $y^2 = 4ax$  and  
 $x^2 = 4ay$ .

- (28) Evaluate  $\iiint (z^5 + 5) dx dy dz$  over the sphere  
 $x^2 + y^2 + z^2 = 1$ .

- (29) Evaluate  $\int_0^1 \int_0^2 (x+y) dx dy$

- (30) Change the order of integration in  $\int_0^1 \int_{\sqrt{x}}^1 e^{xy} dx dy$   
& hence find it's value